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Qst.No.	Scoring Indicators	Split up score	Subtotal	Total
	PART-A			
I 1.	$(\sin A+\cos A)^2 = \sin^2 A+\cos^2 A+2\sin A\cos A$	1		2
	$= 1+2\sin A\cos A$	1		
2.	$\sin 3A = 3\sin A-4\sin^3 A = 3a-4a^3$	1+1		2
3.	$\lim\limits_{\theta\to 0}\left(\frac{\sin 5\theta}{\theta}\right) = \lim\limits_{\theta\to 0}\left(\frac{\sin 5\theta}{5\theta}\right)\cdot 5 = 5$	1+1		2
4.	$\frac{d}{dx}\left(e^{\sqrt{x}}\right) = e^{\sqrt{x}}\cdot\frac{d}{dx}(\sqrt{x}) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$	1+1		2
5.	$V=\frac{4}{3}\pi r^3 \quad \therefore \frac{dV}{dr} = \frac{4}{3}\pi\cdot 3r^2 = 4\pi r^2$	1+1		2
II	PART-B			
1.	$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} = \frac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$	1		
	$= \frac{\tan\theta+\sec\theta-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{\tan\theta-\sec\theta+1}$	1		6
	$= \frac{(\sec\theta+\tan\theta)(1-\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$	1		
	$= \sec\theta+\tan\theta$	1		
	$= \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$	1		
	$= \frac{1+\sin\theta}{\cos\theta}$	1		

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<table>
<tr><th>Qst.No.</th><th>Scoring Indicators</th><th>Split up score</th><th>Subtotal</th><th>Total</th></tr>
<tr><td>II 2.</td><td>$\tan 75 = \tan(45+30) = \dfrac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$
$= \dfrac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$
$\cot 75 = \dfrac{1}{\tan 75} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$
$\therefore \tan 75 + \cot 75 = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} + \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$
$= \dfrac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = 2$</td><td>1+1

1

1

1

1</td><td></td><td>6</td></tr>
<tr><td>II 3.</td><td>$A = \cos^{-1}\left[\dfrac{b^2+c^2-a^2}{2bc}\right] = \cos^{-1}\left(\dfrac{68}{70}\right) = 34^\circ 03'$
$B = \cos^{-1}\left[\dfrac{a^2+c^2-b^2}{2ac}\right] = \cos^{-1}\left(\dfrac{40}{56}\right) = 44^\circ 25'$
$C = 180 - (A+B)$
$= 180 - 78^\circ 28'$
$= 101^\circ 32'$</td><td>1+1

1+1

1

1</td><td></td><td>6</td></tr>
</table>

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II 4.	$y = x^n \;;\; y + \Delta y = (x+\Delta x)^n$	1		
	$\Delta y = (x+\Delta x)^n - x^n$	1		
	$\dfrac{\Delta y}{\Delta x} = \dfrac{(x+\Delta x)^n - x^n}{\Delta x}$	1		6
	$\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{(x+\Delta x)^n - x^n}{\Delta x}$	1		
	$= \lim\limits_{\Delta x \to 0} \dfrac{(x+\Delta x)^n - x^n}{x+\Delta x - x}$	1		
	$\therefore \dfrac{dy}{dx} = n x^{n-1}$ [algebraical limit]	1		
II 5.	$x = 3\cos t - \cos^3 t \;;\; \dfrac{dx}{dt} = -3\sin t + 3\cos^2 t \sin t$	1		
	$y = 3\sin t - \sin^3 t ; \dfrac{dy}{dt} = 3\cos t - 3\sin^2 t \cos t$	1		
	$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{-3\sin t\,(1-\cancel{3}\cos^2 t)}{3\cos t\,(1-\sin^2 t)}$	1		6
	$= -\dfrac{\sin t \cdot \sin^2 t}{\cos t \cos^2 t}$	1		
	$= \dfrac{-\sin^3 t}{\cos^3 t}$	1		
	$= -\cot^3 t$	1		

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Qst.No.	Scoring Indicators	Split up score	Subtotal	Total
II 6.	$3x^2+3y^2y' = 3a(xy'+y)$	2		
	$y'(3y^2-3ax) = 3ay-3x^2$	1		
	$y' = \dfrac{ay-x^2}{y^2-ax}$	1		6
	If the tangents are parallel to x-axis, $\dfrac{dy}{dx}=0$	1		
	ie, $ay=x^2$	1		
II 7.	$P = 2x+2y = 100 \Rightarrow x+y=50$	1		
	$A = xy = x(50-x) = 50x-x^2$	1		
	At maximum, $\dfrac{dA}{dx}=0 \Rightarrow 50-2x=0 \Rightarrow x=25$	1		
	$\dfrac{d^2A}{dx^2} = -2<0$	1		6
	$\therefore$ Area is maximum at $x=25$	1		
	$\therefore x=25,\ y=25$	1		
	PART-C Unit-1			
III a.	$\dfrac{\sec\theta}{\sec\theta+1}+\dfrac{\sec\theta}{\sec\theta-1} = \dfrac{\sec\theta(\sec\theta-1)+\sec\theta(\sec\theta+1)}{\sec^2\theta-1}$	1		5
	$=\dfrac{2\sec^2\theta}{\tan^2\theta} = \dfrac{2/\cos^2\theta}{\sin^2\theta/\cos^2\theta}$	2+1		
	$= 2\operatorname{cosec}^2\theta$	1		

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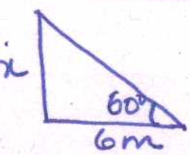
III b	$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$	1		
	$= \dfrac{\frac{1}{11}+\frac{5}{6}}{1-\frac{1}{11}\cdot\frac{5}{6}} = \dfrac{\frac{61}{66}}{\frac{61}{66}}$	1+1		5
	$=1$	1		
	$\Rightarrow \alpha+\beta = \tan^{-1}(1) = 45^\circ$	1		
III c.	$\tan(45-A) = \dfrac{\tan 45 - \tan A}{1+\tan 45 \tan A}$	1		
	$= \dfrac{1-\tan A}{1+\tan A} = \dfrac{1-\frac{\sin A}{\cos A}}{1+\frac{\sin A}{\cos A}}$	1+1		5
	$= \dfrac{\frac{\cos A - \sin A}{\cos A}}{\frac{\cos A + \sin A}{\cos A}} = \dfrac{\cos A - \sin A}{\cos A + \sin A}$	1+1		
IV a.	Let x be the height of the pole. 	1+1		
	$\tan 60^\circ = \dfrac{x}{6}$	1		5
	$\sqrt{3} = \dfrac{x}{6}$	1		
	$x = 6\sqrt{3}$ m	1		

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Qst.No.	Scoring Indicators	Split up score	Subtotal	Total
IV b.	$\sec\theta = \frac{-13}{12}$; $\cos\theta = \frac{-12}{13}$	1		
	$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{144}{169}} = \frac{-5}{13}$ (since θ lies in 3^{rd} quadrant)	1		5
	$\operatorname{cosec}\theta = \frac{-13}{5}$; $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{12}$	1+1		
	$\cot\theta = \frac{12}{5}$	1		
IV c.	$\sin\left(\frac{\pi}{3}+A\right) - \sin\left(\frac{\pi}{3}-A\right)$			
	$= \sin\frac{\pi}{3}\cos A + \cos\frac{\pi}{3}\sin A - \sin\frac{\pi}{3}\cos A + \cos\frac{\pi}{3}\sin A$	2		
	$= 2\cos\frac{\pi}{3}\sin A = 2\cdot\frac{1}{2}\sin A$	1+1		5
	$= \sin A$	1		
	Unit-II			
V a.	$\frac{\sin 3A}{\sin A} + \frac{\cos 3A}{\cos A} = \frac{3\sin A - 4\sin^3 A}{\sin A} + \frac{4\cos^3 A - 3\cos A}{\cos A}$	1+1		
	$= 3 - 4\sin^2 A + 4\cos^2 A - 3$	1		
	$= 4(\cos^2 A - \sin^2 A)$	1		5
	$= 4\cos 2A$	1		

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Qst.No.	Scoring Indicators	Split up score	Subtotal	Total
V b.	$\sin 20 \sin 40 \sin 80 = \sin 20 \cdot \frac{-1}{2}[\cos 120 - \cos 40]$	1		
	$= \frac{-1}{2}\sin 20\left[-\frac{1}{2} - \cos 40\right]$			
	$= \frac{1}{4}\sin 20 + \frac{1}{2}\sin 20 \cos 40$	1		5
	$= \frac{1}{4}\sin 20 + \frac{1}{2}\cdot\frac{1}{2}[\sin 60 - \sin 20]$	1		
	$= \cancel{\frac{1}{4}\sin 20} + \frac{\sqrt{3}}{8} - \cancel{\frac{1}{4}\sin 20}$	1		
	$\therefore \sin 20 \sin 40 \sin 60 \sin 80 = \frac{\sqrt{3}}{8}\cdot\frac{\sqrt{3}}{2} = \frac{3}{16}$	1		
V c.	$abc(\cot A + \cot B + \cot C) = abc\left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}\right)$	1		
	$= \frac{a}{\sin A}\cdot bc\cos A + \frac{b}{\sin B}\cdot ac\cos B + \frac{c}{\sin C}\cdot ab\cos C$	1		
	$= 2R\cdot bc\cos A + 2R\cdot ac\cos B + 2R\cdot ab\cos C$	1		5
	$= R[b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2]$	1		
	$= R[a^2+b^2+c^2]$	1		
VI a.	$C = 180 - (A+B) = 77°$	1		
	$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{a}{\sin 35} = \frac{25}{\sin 77} \Rightarrow a = 14.71$	1+1		5
	$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{b}{\sin 68} = \frac{25}{\sin 77} \Rightarrow b = 23.78$	1+1		

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VI b.	$\sin 50 - \sin 70 + \sin 10 = 2\cos\frac{120}{2}\sin\frac{-20}{2} + \sin 10$	1		
	$= 2\cos 60 \cdot -\sin 10 + \sin 10$	1		
	$= -2\cdot\frac{1}{2}\sin 10 + \sin 10$	1		5
	$= -\sin 10 + \sin 10 = 0$	2		
VI c.	Consider $\tan(50-40) = \frac{\tan 50 - \tan 40}{1+\tan 50 \tan 40}$	1		
	$\tan 10 = \frac{\tan 50 - \tan 40}{1+\tan 50 \tan(90-50)}$	1		
	$= \frac{\tan 50 - \tan 40}{1+\tan 50 \cot 50}$	1		5
	$2\tan 10 = \tan 50 - \tan 40$	1		
	$2\tan 10 + \tan 40 = \tan 50$	1		
	Unit-III			
VII a	$\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{\sin x \frac{d(\cos x)}{dx} - \cos x \frac{d(\sin x)}{dx}}{\sin^2 x}$	1		
	$= \frac{\sin x \cdot -\sin x - \cos x \cdot \cos x}{\sin^2 x}$	1		5
	$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}$	1+1		
	$= -\mathrm{cosec}^2 x$	1		

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VII. b. i)	$\lim \left(\frac{x^3-8}{x^2-4}\right) = \lim \left(\frac{\frac{x^3-2^3}{x-2}}{\frac{x^2-2^2}{x-2}}\right) =$	1		
	$= \lim \left(\frac{x^3-2^3}{x-2}\right) \div \lim \left(\frac{x^2-2^2}{x-2}\right)$	1	3	3
	$= 3 \cdot 2^2 \div 2 \cdot 2^1 = 3$	1		5
ii)	$\lim_{\theta \to 0} \frac{\tan 3\theta}{\theta} = \lim_{\theta \to 0} \frac{\sin 3\theta}{\theta \cdot \cos 3\theta} = \lim_{\theta \to 0} \frac{\sin 3\theta}{\theta} \cdot \lim_{\theta \to 0} \frac{1}{\cos 3\theta}$	1		
	$= 1 \cdot 1 = 1$	1	2	2
VII. c. i)	$y = x e^x \sin^{-1} x$; $\frac{dy}{dx} = x e^x \frac{d}{dx}(\sin^{-1} x) + x \sin^{-1} x \cdot \frac{d}{dx}(e^x) + e^x \sin^{-1} x \frac{d}{dx}(x)$	1	2	2
	$= \frac{x e^x}{\sqrt{1-x^2}} + x \sin^{-1} x \, e^x + e^x \sin^{-1} x$	1		
ii)	$y = \log \sin(x^2+a^2)$			5
	$\frac{dy}{dx} = \frac{1}{\sin(x^2+a^2)} \cdot \frac{d}{dx}\left(\sin(x^2+a^2)\right)$	1		
	$= \frac{1}{\sin(x^2+a^2)} \cdot \cos(x^2+a^2)(2x)$	1	3	3
	$= 2x \cot(x^2+a^2)$	1		

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VIII a.	$\frac{dx}{dt} = e^t \cdot -\sin t + \cos t\, e^t$	2		
	$\frac{dy}{dt} = e^t \cos t + \sin t\, e^t$	2		5
	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^t(\cos t - \sin t)}{e^t(\cos t + \sin t)}$			
	$= \frac{\cos t - \sin t}{\cos t + \sin t}$	1		
VIII b.	$(1-x^2)y'' - xy' = ?$ $\quad y' = \frac{1}{\sqrt{1-x^2}}$	1		
	$y'' = \frac{+x}{(1-x^2)^{3/2}}$	1		
	$(1-x^2)y'' - xy' = (1-x^2) \cdot \frac{+x}{(1-x^2)\sqrt{1-x^2}} - x \cdot \frac{1}{\sqrt{1-x^2}}$	1		5
	$= \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = 0$	1+1		
VIII c.	i) $\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$			
	$= \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}$	1+1	3	
	$= \sec x$	1		
	ii) $\frac{dy}{dx} = \frac{(x^3-1)^2 \cdot \frac{d}{dx}(\cot 11x) - \cot 11x \cdot \frac{d}{dx}(x^3-1)^2}{(x^3-1)^4}$	1		5
	$= \frac{-(x^3-1)^2 \cdot 11 \operatorname{cosec}^2 x - 2(x^3-1) \cdot 3x^2 \cdot \cot 11x}{(x^3-1)^4}$	1	2	

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	UNIT-IV			
IX a.	$\frac{dy}{dx} = 6x^2 - 18x + 12$	1		
	Since the tangent is parallel to x-axis,			
	slope of tangent = slope of x-axis			5
	ie, $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 18x + 12 = 0$	1		
	$\Rightarrow x^2 - 3x + 2 = 0$	1		
	$\Rightarrow (x-1)(x-2) = 0$	1		
	$\Rightarrow x = 1, 2.$	1		
IX b.	Velocity $= \frac{dx}{dt} = -12\sin 3t + 15\cos 3t$	1		
	Acceleration $= \frac{d^2x}{dt^2} = -36\cos 3t + 45 \cdot -\sin 3t$	1		
	$= -9(4\cos 3t + 5\sin 3t)$	1		5
	$= -9x.$			
	ie, $a = -9x$	1		
	$\Rightarrow$ acceleration $\propto$ displacement	1		

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IX c.	$y = 2x^3 - 3x^2 - 36x + 10$			
	$\frac{dy}{dx} = 6x^2 - 6x - 36$	1		
	At maxima or minima, $\frac{dy}{dx} = 0$			
	$x^2 - x - 6 = 0$	1		
	$(x-3)(x+2) = 0 \Rightarrow x = -2, 3.$	1		
	$\frac{d^2y}{dx^2} = 12x - 6$			5
	At $x = -2$, $\frac{d^2y}{dx^2} < 0$			
	At $x = 3$, $\frac{d^2y}{dx^2} > 0$.	1		
	$\therefore$ y is minimum at $x=3$ and the minimum value $= 54 - 27 - 108 + 10$ $= -71$	1		
X a.	$V = lbh = (8-2x)(8-2x)x$			
	$= 4x^3 - 32x^2 + 64x$	1		
	At a maxima or minima, $\frac{dV}{dx} = 0$			
	$\cancel{x^3 - 8x^2 + 16}$ $3x^2 - 16x + 16 = 0$.	1	5	
	$x = 4, 4/3$.	1		
	$\frac{d^2V}{dx^2} = 6x - 16$. $x = 4$ is not admissible since length is 8 cm.	1		
	size of the square to be cut off $= 4/3$ cm	1		

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X b.	Volume of the spherical balloon $= \frac{4}{3}\pi r^3$	1		
	$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 25$	1		
	$4\pi \times 15^2 \times \frac{dr}{dt} = 25$.	1		5
	$\frac{dr}{dt} = \frac{25}{4\pi \times 15^2} = \frac{1}{36\pi}$ cm/sec.	1+1		
X c.	Slope of the tangent $= \frac{dy}{dx}$.			
	$2y\frac{dy}{dx} = 4a$.	1		
	$\frac{dy}{dx} = \frac{2a}{y}$	1		
	$\frac{dy}{dx}$ at $(a, 2a) = 1$.	1		
	Equation of the tangent to the curve is $y - y_1 = \frac{dy}{dx}(x - x_1)$			5
	$y - 2a = x - a$.	1		
	$x - y + a = 0$	1		