1002 (S) APRIL 2022 ①

I PART-A

1. LHS $= (1 - \sin^2 A) - \sin^2 A \rightarrow (1)$

$= 1 - 2\sin^2 A \rightarrow (1)$

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2. $\cos 3A = 4\cos^3 A - 3\cos A \rightarrow (1)$

$= -\dfrac{44}{125} \rightarrow (1)$

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3. Area $= \dfrac{1}{2} bc \sin A \rightarrow (1)$

$= \dfrac{3}{2} \rightarrow (1)$

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4. $\dfrac{dy}{dx} = x \cdot \dfrac{d}{dx}(\tan x) + \tan x \cdot \dfrac{d}{dx}(x) \rightarrow (1)$

$= x\sec^2 x + \tan x \rightarrow (1)$

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5. Velocity $v = \dfrac{ds}{dt} = 2t - 3 \rightarrow (1)$

Accelaration, $a = \dfrac{d^2 s}{dt^2} = 2 \rightarrow (1)$

PART-B

II
1. Let $4\cos\theta + 3\sin\theta = R\sin(\theta + \alpha)$

$= R[\sin\theta\cos\alpha + \cos\theta\sin\alpha] \rightarrow (1)$

$$\left.\begin{aligned} \therefore 4 &= R\sin\alpha \quad — (1) \\ 3 &= R\cos\alpha \quad — (2) \end{aligned}\right\} \rightarrow (2)$$

$\therefore R = 5 \rightarrow (1)$

$\alpha = 53^\circ 8' \rightarrow (1)$

$\therefore\ 4\cos\theta + 3\sin\theta = 5\sin(\theta + 53^\circ 8') \rightarrow (1)$ 2

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2. $LHS = \frac{1}{2}\left[2\cos 20 \cos 40\right]\cos 80$

$= \frac{1}{2}\left[\cos 60 + \cos 20\right]\cos 80 \longrightarrow (1)$

$= \frac{1}{4}\cos 80 + \frac{1}{4}\left(2\cos 20 \cos 80\right) \longrightarrow (1)$

$= \frac{1}{4}\cos 80 + \frac{1}{4}\left(\cos 100 + \cos 60\right) \longrightarrow (1)$

$= \frac{1}{4}\cos 80 + \frac{1}{4}\cos 100 + \frac{1}{8} \longrightarrow (1)$

$= \frac{1}{4}\cos 80 - \frac{1}{4}\cos 80 + \frac{1}{8} \longrightarrow (1)$

$= \frac{1}{8} \longrightarrow (1)$

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3. $RHS = abc\cot A + abc\cot B + abc\cot C$

$= \frac{a}{\sin A} bc\cos A + \frac{b}{\sin B} ac\cos B + \frac{c}{\sin C} ab\cos C \rightarrow 1$

$= 2Rbc\cos A + 2Rac\cos B + 2Rab\cos C$

$= R\left[2bc\cos A + 2ac\cos B + 2ab\cos C\right] \rightarrow 1$

$= R\left[b^2+c^2-a^2 + a^2+c^2-b^2 + a^2+b^2-c^2\right] \rightarrow 3$

$= R\left\{a^2+b^2+c^2\right\} \rightarrow 1$

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4. Let $y = \cos x$. Then $y + \Delta y = \cos(x+\Delta x) \rightarrow 1$

i.e. $\Delta y = \cos(x+\Delta x) - \cos x$

$\therefore\ \frac{\Delta y}{\Delta x} = \frac{\cos(x+\Delta x) - \cos x}{\Delta x} \longrightarrow 1$

PAGE NUMBER 3 NOT FOUND IN MASTER COPY

ie $y+4=-\frac{4}{3}(x-3) \rightarrow 1$

$\therefore 4x+3y=0 \rightarrow 1$

7. $\cos 570 = -\cos 30 = -\sqrt{3}/2 \rightarrow 1$

$\sin 510 = \sin 150 = 1/2 \rightarrow 1$

$\sin 330 = -\sin 30 = -1/2 \rightarrow 1$

$\cos 390 = \cos 30 = \sqrt{3}/2 \rightarrow 1$

$LHS = -\frac{\sqrt{3}}{2}\times\frac{1}{2} - {}^{-}1/2 \times \frac{\sqrt{3}}{2} = 0 \rightarrow 2$

PART - C

UNIT - I

III (A) $LHS = \frac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)\cos\theta} \rightarrow 1$

$= \frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{(1+\sin\theta)\cos\theta} \rightarrow 1$

$= \frac{2(1+\sin\theta)}{(1+\sin\theta)\cos\theta} \rightarrow 1$

$= \frac{2}{\cos\theta} = 2\sec\theta = RHS \rightarrow 1$

$\rightarrow 1$

(b) $\left\{ \cos A = \frac{15}{17} \qquad \cos B = \frac{4}{5} \right\} \rightarrow 1$

$\sin(A-B) = \sin A \cos B - \cos A \sin B \rightarrow 1$

$$= \frac{8}{17} \times \frac{4}{5} - \frac{15}{17} \times \frac{3}{5}$$

$$= \frac{32}{85} - \frac{45}{85} = \frac{-13}{85} \longrightarrow 1$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \longrightarrow 1$$

$$= \frac{15}{17} \times \frac{4}{5} - \frac{8}{17} \times \frac{3}{5}$$

$$= \frac{60}{85} - \frac{24}{85} = \frac{36}{85} \longrightarrow 1$$

(c)

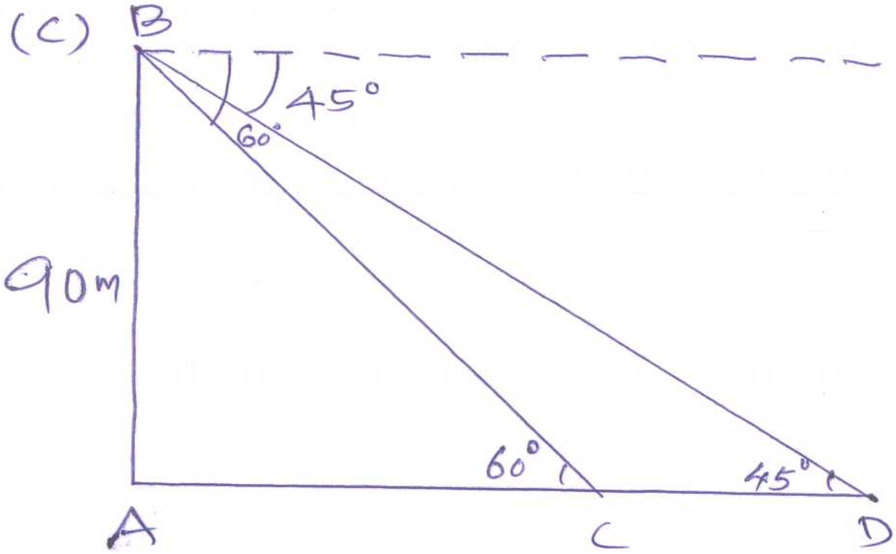

Figure. $\longrightarrow$ 1

Let CD be the distance the two boats.

$CD = AD - AC.$

$$\tan 60 = \frac{90}{AC} \cdot \text{ ie } AC = \frac{90}{\sqrt{3}} \longrightarrow 1$$

$$\tan 45 = \frac{90}{AD} \cdot \text{ ie } AD = 90 \longrightarrow 1$$

$$\therefore CD = 90 - \frac{90}{\sqrt{3}} \longrightarrow 1$$

$$= \underline{\underline{38.04m}} \longrightarrow 1$$

IV (A) LHS = $\dfrac{\text{Cosec}A(\text{Cosec}A+1) + \text{Cosec}A(\text{Cosec}A-1)}{(\text{Cosec}A-1)(\text{Cosec}A+1)}$ ⑥ $\longrightarrow$ ①

$= \dfrac{2\text{Cosec}^2 A}{\text{Cosec}^2 A - 1} \longrightarrow$ ①

$= \dfrac{2\text{Cosec}^2 A}{\text{Cot}^2 A} \longrightarrow$ ①

$= \dfrac{2}{\text{Cos}^2 A} \longrightarrow$ ①

$= 2\text{Sec}^2 A \longrightarrow$ ①

(b) $\text{Cos}A = \dfrac{\sqrt{21}}{5} \longrightarrow$ ①

$\text{Sin}2A = 2\text{Sin}A\,\text{Cos}A \longrightarrow$ ①

$= 2 \cdot \dfrac{2}{5} \cdot \dfrac{\sqrt{21}}{5} = \dfrac{4\sqrt{21}}{25} \longrightarrow$ ①

$\text{Cos}2A = \left.\begin{array}{l} 1-2\text{Sin}^2 A \\ \text{Cos}^2 A - \text{Sin}^2 A \\ 2\text{Cos}^2 A - 1 \end{array}\right\} \longrightarrow$ ①

$= \dfrac{-17}{25} \longrightarrow$ ①

(c) $\tan 75 = \tan(45+30) = \dfrac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} \longrightarrow$ ①

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \longrightarrow$ ①

$\text{Cot}\,75 = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ — ①

$\therefore \tan 75 + \cot 75 = \dfrac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} \longrightarrow$ ①

$= 4 \longrightarrow$ ①

UNIT – II

V. (a) $LHS = \dfrac{(\sin A + \sin 5A) + \sin 3A}{(\cos A + \cos 5A) + \cos 3A} \longrightarrow 1$

$= \dfrac{2\sin 3A \cos 2A + \sin 3A}{2\cos 3A \cos 2A + \cos 3A} \longrightarrow 2$

$= \dfrac{\sin 3A (2\cos 2A + 1)}{\cos 3A (2\cos 2A + 1)} \longrightarrow 1$

$= \dfrac{\sin 3A}{\cos 3A} = \tan 3A = RHS \longrightarrow 1$

(b) $LHS = (\sin 7\theta + \sin\theta) + (\sin 5\theta + \sin 3\theta) \longrightarrow 1$

$= 2\sin 4\theta \cos 3\theta + 2\sin 4\theta \cos\theta \longrightarrow 2$

$= 2\sin 4\theta (\cos 3\theta + \cos\theta) \longrightarrow 1$

$\left.\begin{array}{l} = 2\sin 4\theta \cdot 2\cos 2\theta \cdot \cos\theta \\ = 4\cos\theta \cos 2\theta \sin 4\theta \end{array}\right\} \longrightarrow 1$

(c) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} \longrightarrow 1 \quad = \dfrac{21}{24} = 0.875$

$\therefore A = 28^\circ 57' \longrightarrow 1$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac} \longrightarrow 1 \quad = \dfrac{11}{16} = 0.6875$

$\therefore B = 46^\circ 34' \longrightarrow 1$

$C = 180 - (A + B) = 104^\circ 29' \longrightarrow 1$

OR

VI (a) $LHS = (\sin 50 - \sin 70) + \sin 10 \longrightarrow 1$

$= -2\cos 60 \sin 10 + \sin 10 \longrightarrow 2$

$= -2 \cdot \frac{1}{2} \cdot \sin 10 + \sin 10 \longrightarrow 1$

$= 0 \longrightarrow (1)$

(b) $LHS = (\cos 55 + \cos 65) + \cos 175 \longrightarrow (1)$

$= 2\cos 60 \cos 5 + \cos 175 \longrightarrow (1)$

$= 2 \times \frac{1}{2} \cos 5 + \cos 175 \longrightarrow (1)$

$= \cos 5 + \cos(180 - 5) \longrightarrow (1)$

$= \cos 5 - \cos 5 = 0 \longrightarrow (1)$

(c) $\tan \frac{B-A}{2} = \frac{b-a}{b+a} \cot \frac{C}{2} \longrightarrow (1)$

$\frac{B-A}{2} = 40^\circ 44' \longrightarrow (1)$ $\therefore B - A = 80' 88'$

$B + A = 150'$

$2B = 230^\circ 88'$

$\therefore B = 115^\circ 44' \longrightarrow (1)$

$A = 34^\circ 16'. \longrightarrow (1)$

$\left. c^2 = a^2 + b^2 - 2ab\cos C \right\} \longrightarrow (1)$

$\therefore c = 4.44$ cm.

UNIT - III

VII (a) (i) $\lim_{x \to 0} \frac{\sin 3x \cdot \cos x}{x} = \lim_{x \to 0} 3 \times \frac{\sin 3x}{3x} \times \lim_{x \to 0} \cos x \longrightarrow (1)$

$= 3 \times 1 \times \cos 0 \longrightarrow (1)$

$= 3 \longrightarrow (1)$

(ii) $\lim_{x \to \infty} \frac{3x^2 - x + 1}{2x^2 + 2x + 1} = \lim_{x \to \infty} \frac{x^2\left(3 - \frac{1}{x} + \frac{1}{x^2}\right)}{x^2\left(2 + \frac{2}{x} + \frac{1}{x^2}\right)} \longrightarrow (1)$

$= \lim_{x \to \infty} \frac{3 - \frac{1}{x} + \frac{1}{x^2}}{2 + \frac{2}{x} + \frac{1}{x^2}} \longrightarrow (1)$ $= \frac{3}{2}$ — (1)

(b) $\frac{dx}{d\theta} = a\sec\theta\tan\theta \rightarrow 1$ $\quad \frac{dy}{d\theta} = b\sec^2\theta \rightarrow 1$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b}{a}\frac{\sec\theta}{\tan\theta} \rightarrow 1 \quad = \frac{b}{a}\operatorname{cosec}\theta.$$

$\hookrightarrow 1$

(c) $\frac{dy}{dx} = nAe^{nx} - nBe^{-nx} \rightarrow 2$

$$\frac{d^2y}{dx^2} = n^2Ae^{nx} + n^2Be^{-nx} \rightarrow 2$$

$$\left.\begin{aligned} &= n^2(Ae^{nx} + Be^{-nx}) \\ &= n^2y \\ \therefore\ &\frac{d^2y}{dx^2} - n^2y = 0 \end{aligned}\right\} \rightarrow 1$$

OR

VIII (a) (i) $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}}{x^2} \rightarrow 1$

$$= \lim_{x\to 0} 2\cdot \frac{\sin\frac{x}{2}}{\frac{x}{2}} \cdot \frac{\sin\frac{x}{2}}{\frac{x}{2}} \times \frac{1}{4} \rightarrow 2$$

$$= \frac{1}{2} \times 1 \times 1 = \frac{1}{2} \rightarrow 1$$

(ii) $\lim_{x\to 1} \frac{x^3-1}{x-1} = 3\cdot 1^{3-1} \rightarrow 1$

$$= 3 \rightarrow 1$$

(b) $\frac{dy}{dx} = \frac{1}{\sec x - \tan x} \times (\sec x\tan x - \sec^2 x) \rightarrow 2$

$$= \frac{1}{\sec x - \tan x} \times \sec x(\tan x - \sec x) \rightarrow 1$$

$$= -\sec x \rightarrow 1$$

(10)

(c) $\frac{dy}{dx} = am\cos mx \rightarrow (1)$ $\quad \frac{d^2y}{dx^2} = -am^2\sin mx \rightarrow (1)$

$= -m^2(a\sin mx) \rightarrow (1)$

$= -m^2y \rightarrow (1)$

$\therefore \frac{d^2y}{dx^2} + m^2y = 0 \rightarrow (1)$

UNIT IV

IX (a) $\frac{dy}{dx} = 6x+1 \quad \therefore m = 7 \longrightarrow (1)$

Eqn. of the tangent is $y-y_1 = m(x-x_1) \rightarrow (1)$

ie $7x-y-5=0 \longrightarrow (1)$

Eqn. of the normal is $y-y_1 = -\frac{1}{m}(x-x_1) \rightarrow (1)$

ie $x+7y-15=0 \longrightarrow (1)$

(b) Area, $A = \pi r^2 \rightarrow (1)$ $\quad \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \rightarrow (2)$

$\therefore \frac{dr}{dt} = \frac{\frac{dA}{dt}}{2\pi r} \rightarrow (1) = \frac{6}{2\pi \times 2}$

$= \frac{3}{2}\pi$ cm/min. $\longrightarrow (1)$

(c) Let x and y be the dimensions of the rectangle.

$\therefore P = 2x+2y$ ie $\frac{P}{2} = x+y$

$\therefore y = \frac{P-2x}{2} \longrightarrow (1)$

Area, $A = xy \quad = x\left(\frac{P-2x}{2}\right) = \frac{1}{2}(Px - 2x^2) \rightarrow (1)$

$\frac{dA}{dx} = 0 \Rightarrow \frac{1}{2}(P-4x) = 0 \longrightarrow (1)$

$\therefore x = \frac{P}{4}; \quad y = \frac{P}{4}$