

**DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/
MANAGEMENT/COMMERCIAL PRACTICE, NOVEMBER – 2022**

ENGINEERING MATHEMATICS – I

[Maximum Marks: 100]

[Time: 3 Hours]

PART-A

[Maximum Marks: 10]

I. (Answer *all* questions in one or two sentences. Each question carries 2 marks)

1. If $\sin\theta = \frac{1}{2}$, θ is acute, Find $\cos\theta$.
2. Find area of triangle ABC, if $b = 3\text{cm}$, $c = 2\text{cm}$, $A = 30^\circ$
3. Evaluate $\lim_{x \rightarrow 1} \frac{2x+3}{4x-1}$.
4. Find $\frac{dy}{dx}$ if $y = 3 \cos x - 4 \tan x$.
5. Find rate of change of volume of a sphere with respect to its radius. (5 x 2 = 10)

PART-B

[Maximum Marks: 30]

II. (Answer any *five* of the following questions. Each question carries 6 marks)

1. Find the value of $\tan 75$ without using tables and show that $\tan 75 + \cot 75 = 4$.
2. Prove that $\frac{\sin(180+A) \cdot \cos(90-A) \cdot \tan(270+A)}{\sec(540-A) \cdot \cos(360+A)} = -\sin A \cos A$.
3. Prove that $\cos 20 \cdot \cos 40 \cdot \cos 60 \cdot \cos 80 = \frac{1}{16}$.
4. Solve ΔABC if $a = 2\text{cm}$, $b = 3\text{cm}$, $c = 4\text{cm}$.
5. Differentiate $\sin x$ using method of first principle.
6. If $y = a \cos \log x + b \sin \log x$, Prove that $x^2 y'' + xy' + y = 0$.
7. Find equation of tangent and normal to the curve $y = 3x^2 + x - 2$ at (1,2). (5 x 6 = 30)

PART-C

[Maximum Marks: 60]

(Answer *one* full question from each Unit. Each full question carries 15 marks)

UNIT - I

- III (a)** Show that $\frac{\sin A}{1-\cos A} + \frac{1-\cos A}{\sin A} = 2 \operatorname{cosec} A$ (5)

- (b) The horizontal distance between two towers is 60m and the angle of depression of the first tower as seen from the second which is in 150m height is 30° . Find the height of first tower. (5)
- (c) Express $x = 3 \sin\theta + 4 \cos\theta$ in the form $R \sin(\theta + a)$, a is acute. (5)

OR

- IV (a) If $\tan A = \frac{3}{4}$, $\sin B = \frac{5}{13}$, A lies in 3rd quadrant and B lies in 1st quadrant.
Find $\sin(A - B)$ and $\cos(A + B)$. (6)
- (b) Prove that $\sqrt{\frac{1-\sin A}{1+\sin A}} = \sec A - \tan A$. (4)
- (c) If $\theta = 30^\circ$, verify that $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$. (5)

UNIT - II

- V (a) Prove that $\frac{\sin 2A + \sin 5A - \sin 3A}{\cos 2A + \cos 5A + \cos 3A} = \tan 2A$. (5)
- (b) Prove that $R(a^2 + b^2 + c^2) = abc(\cot A + \cot B + \cot C)$. (5)
- (c) Prove that $\sin A + \sin 3A + \sin 5A + \sin 7A = 4 \cos A \cdot \cos 2A \cdot \sin 4A$ (5)

OR

- VI (a) Prove that $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = 2$. (5)
- (b) Solve $\triangle ABC$ if $A=35^\circ, B = 68^\circ$ and $c = 25\text{cm}$. (5)
- (c) Prove that $\sin 50 - \sin 70 + \cos 80 = 0$. (5)

UNIT- III

- VII (a) Evaluate (i) $\lim_{x \rightarrow \infty} \frac{x^3 - 7x^2 - 2}{4x^3 - 2x - 5}$ (3)
- (ii) $\lim_{\theta \rightarrow 0} \frac{\sin 3\theta \cdot \cos \theta}{\theta}$. (2)
- (b) Find $\frac{dy}{dx}$ if $x = a \sec \theta, y = b \tan \theta$. (4)
- (c) Find $y = \frac{e^{2x} \cdot \tan^{-1} 3x}{\sqrt{x}}$ (6)

OR

- VIII (a) Evaluate $\frac{dy}{dx}$ if $y = (1 + x^2)^{10} \sin^2 x$. (5)
- (b) Find $\frac{dy}{dx}$ if $ax^2 + by^2 + 2gx + 2fy + c = 0$. (5)
- (c) Find $\frac{dy}{dx}$ if $x = a(\theta - \sin \theta), y = a(1 - \cos \theta)$. (5)

UNIT - IV

- IX (a) For what values of x is the tangent to the curve $\frac{x}{x^2+1}$ is Parallel to x -axis. (5)
- (b) If displacement of a particle is given by $s = a \cos nt + b \sin nt$, Prove that acceleration varies as displacement. (5)
- (c) Deflection of a beam is given by $y = 2x^3 - 9x^2 + 12x$. Find maximum deflection. (5)

OR

- X (a) A balloon is spherical in shape. Gas is escaping from it at the rate of 20cc per second. How fast is the surface area shrinking when radius is 15cm? (5)
- (b) The displacement of a particle is given by $s = 2t^3 - 9t^2 + 12t + 6$. Find value of t when acceleration is zero. Find velocity at that time. (5)
- (c) An open box is made out of a square sheet of side 8cm, by cutting off equal squares at each corner and turning up the sides. What size of squares should be cut in order that volume of the box may be the maximum. (5)
