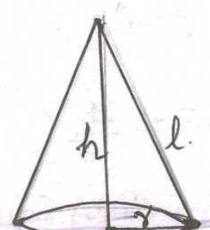
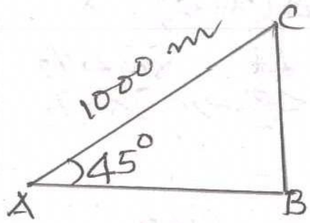
1002A(15)

Revision : 2015		Course Code : 1002	
Course Title : ENGINEERING MATHEMATICS- I			
Qst. No	**Scoring Indicator**	**Split Score**	**Total Score**
I	Part A		
	$\sin \pi/6 = 1/2$ $\quad \cos \pi/3 = \cos 60 = 1/2$		
1.	$\therefore 3\times\frac{1}{2} - 4\left(\frac{1}{2}\right)^3 = \frac{3}{2} - 4\cdot\frac{1}{8}$	1	2
	$= \underline{\underline{1}}$	1	
2.	Applying, cosine rule		
	$c^2 = a^2 + b^2 - 2ab\cos C$		
	$= 25 + 64 - 80\times\frac{1}{2} = 49$	1	2
	$\therefore c = \sqrt{49} = \underline{7}$	1	
3	$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}bc\cdot\frac{a}{2R} = \frac{abc}{4R}$	1	2
	$\therefore abc = 4R\Delta$	1	
4	$\frac{dy}{dx} = x^3\frac{d}{dx}(\tan x) + \tan x\frac{d}{dx}(x^3)$	1	2
	$= x^3\sec^2 x + 3x^2\tan x$	1	
5	$A = \pi r^2$	1	2
	$\frac{dA}{dr} = 2\pi r$	1	
	Part B		
II 1	$\tan 15^\circ = \tan(45-30)$		
	$= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$	1	
	$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	1	
	$\therefore \cot 15^\circ = \frac{1}{\tan 15^\circ} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$	1	6
	$\therefore \tan 15^\circ + \cot 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$	1	
	$= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$	1	
	$= 8/2 = \underline{\underline{4}}$	1	

2.	$\sqrt{3}\cos x + \sin x = R\sin(x+\alpha)$		
	$= R[\sin x \cos\alpha + \cos x \sin\alpha]$	1	
	$\therefore R\sin\alpha = \sqrt{3}$	1	
	$R\cos\alpha = 1$		
	$R^2 = (\sqrt{3})^2 + 1^2 = 4$	1	6
	$R = \pm 2$	1	
	$\alpha = \tan^{-1}(\sqrt{3}) = 60^\circ$	1	
	$\therefore \sqrt{3}\cos x + \sin x = \pm 2\sin(x+60^\circ)$	1	
3.	$\sin 20(\sin 40 \sin 80) = \frac{1}{2}\sin 20[\cos 40 - \cos 120]$	1	
	$= \frac{1}{2}\sin 20(\cos 40 + \frac{1}{2})$	1	
	$= \frac{1}{2}\sin 20 \cos 40 + \frac{1}{4}\sin 20$		
	$= \frac{1}{4}(\sin 60 - \sin 20) + \frac{1}{4}\sin 20$	1	6
	$= \frac{1}{4}(\frac{\sqrt{3}}{2} - \sin 20) + \frac{1}{4}\sin 20$	1	
	$= \frac{\sqrt{3}}{8} - \frac{1}{4}\sin 20 + \frac{1}{4}\sin 20$	1	
	$= \sqrt{3}/8$	1	
4	Applying, cosine formulae		
	$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{58}{70}$	1	
	$\therefore A = \cos^{-1}(0.8286) = 34^\circ 4'$	1	
	$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{40}{56}$	1	6
	$B = \cos^{-1}(0.7143) = 44^\circ 25'$	1	
	$\therefore C = 180 - (A+B)$	1	
	$= 180 - 78^\circ 29' = 101^\circ 31'$	1	
	$A = 34^\circ 4'$, $B = 44^\circ 25'$		
	$C = 101^\circ 31'$		

5.	$y+\Delta y = \sin(x+\Delta x)$	1	
	$\Delta y = \sin(x+\Delta x) - \sin x$	1	
	$= 2\cos(x+\Delta x/2)\,\sin\frac{\Delta x}{2}$	1	
	$\frac{\Delta y}{\Delta x} = \frac{2\cos(x+\Delta x/2)\,\sin\frac{\Delta x}{2}}{2\times\frac{\Delta x}{2}}$	1	
	$\underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x} = \underset{\frac{\Delta x}{2}\to 0}{Lt} \cos(x+\frac{\Delta x}{2}) \times \underset{\frac{\Delta x}{2}\to 0}{Lt} \frac{\sin \Delta x/2}{\Delta x/2}$	1	6
	$\frac{dy}{dx} = \cos x \times 1$	1	
	$\therefore \frac{d}{dx}(\sin x) = \underline{\underline{\cos x}}$		
6.	Velocity, $v = \frac{ds}{dt} = -12\sin 4t + 20\cos 4t$	1	
	Acceleration, $a = \frac{d^2s}{dt^2} = -12\times 4\cos 4t - 20\times 4\sin 4t$	1	
	$\therefore a = -48\cos 4t + 60\sin 4t$	1	6
	$= -16(3\cos 4t + 5\sin 4t)$	1	
	$= -16S$	1	
	$\therefore \underline{a \propto S}$	1	
7.			
	$l^2 = r^2 + h^2$ $\therefore r^2 = l^2 - h^2$	1	
	Volume of a Cone, $V = \frac{1}{3}\pi r^2 h$		
	$\therefore V = \frac{1}{3}\pi(l^2 - h^2)h$	1	
	$\frac{dV}{dh} = \frac{1}{3}\pi l^2 - \pi h^2$	1	6
	$\frac{dV}{dh} = 0 \Rightarrow h^2 = \frac{l^2}{3}$ or $h = l/\sqrt{3}$	1	
	$\frac{d^2V}{dh^2} = -2\pi h = -2\pi l/\sqrt{3} < 0$	1	
	$\therefore$ Max. volume at $h = l/\sqrt{3}$ is $\underline{2\pi l^3/9\sqrt{3}}$	1	

	Part C		
III (1)	LHS $= \cot^2 A - 2\cot A + 1 + \cot^2 A + 2\cot A + 1$	1+1	
	$= \cot^2 A + 1 + \cot^2 + 1$	1	
	$= \operatorname{cosec}^2 A + \operatorname{cosec}^2 A$	1	5
	$= \underline{\underline{2 \operatorname{cosec}^2 A}}$	1	
2	 AB = horizontal distance		
	AC = 1000 m	1	
	$\cos 45^\circ = \frac{AB}{AC}$	1	
	$\frac{1}{\sqrt{2}} = \frac{AB}{1000}$	1	5
	$AB = \frac{1000}{\sqrt{2}} = 500\sqrt{2}$	1	
	$= 707.12$ m		
	Horizontal distance $= \underline{\underline{707.12 \text{ m}}}$	1	
3	$\sin A = \frac{-4}{5}$ $\quad \operatorname{cosec} A = \frac{-5}{4}$	1	
	$\cos A = \frac{-3}{5}$ $\quad \sec A = \frac{-5}{3}$	1+1	5
	$\tan A = \underline{\underline{\frac{4}{3}}}$ $\quad \cot A = \underline{\underline{\frac{3}{4}}}$	1+1	
IV 1	$\cos A = \frac{15}{17}$ $\quad \cos B = \frac{4}{5}$	1	
	$\sin(A-B) = \sin A \cos B - \cos A \sin B$	1	
	$= \frac{8}{17} \cdot \frac{4}{5} - \frac{15}{17} \cdot \frac{3}{5} = \underline{\underline{\frac{-13}{85}}}$	1	5
	$\cos(A+B) = \cos A \cos B - \sin A \sin B$	1	
	$= \frac{15}{17} \frac{4}{5} - \frac{8}{17} \frac{3}{5} = \underline{\underline{\frac{36}{85}}}$	1	

Q. No.		Split Score	Total Score
IV 3.	$\cos(90+A) = -\sin A$		
	$\sec(360+A) = \sec A$	1	
	$\tan(180-A) = -\tan A$		
	$\sec(A-720) = \sec A$	1	
	$\sin(540+A) = -\sin A$	1	5
	$\cot(A-90) = -\tan A$		
	$\therefore LHS = \dfrac{(-\sin A)\sec A(-\tan A)}{\sec A(-\sin A)(-\tan A)}$	1	
	$= \underline{\underline{1}}$	1	
V 1.	$LHS = \dfrac{(\sin 5A+\sin A)+\sin 3A}{(\cos 5A+\cos A)+\cos 3A}$	1	
	$= \dfrac{2\sin 3A\cos 2A+\sin 3A}{2\cos 3A\cos 2A+\cos 3A}$	1 + 1	5
	$= \dfrac{\sin 3A(2\cos 2A+1)}{\cos 3A(2\cos 2A+1)}$	1	
	$= \dfrac{\sin 3A}{\cos 3A} = \underline{\underline{\tan 3A = RHS}}$	1	
2.	$LHS = ab^2\cos A+ac^2\cos A+bc^2\cos B+ba^2\cos B+ca^2\cos C+cb^2\cos C$	1	
	$= ab(b\cos A+a\cos B)+bc(c\cos B+b\cos C)+ac(a\cos C+c\cos A)$	2	5
	$= abc+bc\cdot a+ac\cdot b$	1	
	$= \underline{\underline{3abc}}$	1	

V(3)	Applying cosine formulae		
	$c^2 = a^2+b^2-2ab\cos C$.	1	
	$= 89-40\sqrt{3} = 19.72$		
	$\therefore c = 4.44$	1	
	Applying sine or tangent formula.		
	$\frac{a}{\sin A} = \frac{c}{\sin C}$	1	5
	$\sin A = \frac{a\sin C}{c} = \frac{5\sin 30}{4.44} = 0.563$.		
	$A = \sin^{-1}(0.563) = \underline{\underline{34^\circ 16'}}$	1	
	$B = 180-(A+C) = 180-64^\circ 16'$		
	$= \underline{\underline{115^\circ 44'}}$	1	
$\underline{\text{VI}}$(1)	$LHS = \sin 33^\circ + \cos(90-27)$	1	
	$= \sin 33 + \sin 27$	1	
	$= 2\sin 30\cos 3$.	1	
	$= 2\times\frac{1}{2}\cos 3$	1	5
	$= \underline{\underline{\cos 3}} = RHS$	1	
2)	$LHS = \frac{\cos A}{\sin A} - \frac{\cos 2A}{\sin 2A}$	1	
	$= \frac{\sin 2A\cos A - \cos 2A\sin A}{\sin A\sin 2A}$	1	
	$= \frac{\sin(2A-A)}{\sin A\sin 2A}$	1	
			5
	$= \frac{\sin A}{\sin A\sin 2A}$	1	
	$= \frac{1}{\sin 2A} = \underline{\underline{\operatorname{cosec} 2A}}$	1	

VI (3)	LHS $= 2bc\cos A + 2ca\cos B + 2ab\cos C$	1	
	Applying cosine formula.		
	$a^2 = b^2+c^2-2bc\cos A$		
	IIIly. $2bc\cos A = b^2+c^2-a^2$		
	$2ac\cos B = a^2+c^2-b^2$	2	
	& $2ab\cos C = a^2+b^2-c^2$		5
	$\therefore$ LHS $= b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2$	1	
	$= b^2+c^2+a^2 = \underline{\underline{a^2+b^2+c^2}} =$ RHS	1	
VII (1)	$\lim_{x\to 4} \frac{x^3-4^3}{x^2-4^2} = \lim_{x\to 4}\left(\frac{\frac{x^3-4^3}{x-4}}{\frac{x^2-4^2}{x-4}}\right)$	1	
	$= \frac{\lim_{x\to 4}\frac{x^3-4^3}{x-4}}{\lim_{x\to 4}\frac{x^2-4^2}{x-4}}$	1 1	5
	$= \frac{3\cdot 4^{3-1}}{2\cdot 4^{2-1}} = \frac{3\times 4^2}{2\times 4}$	1	
	$= \underline{\underline{6}}$	1	
2)	$\frac{dx}{d\theta} = a(1+\cos\theta)$	1	
	$\frac{dy}{d\theta} = a\sin\theta$	1	
	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)}$	1	5
	$= \frac{2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2}$	1	
	$= \underline{\underline{\tan\theta/2}}$	1	

VII (3)	$\frac{dy}{dx} = -Ap\sin px + Bp\cos px$	1	
	$\frac{d^2y}{dx^2} = -Ap^2\cos px - Bp^2\sin px$	1	
	$= -p^2(A\cos px + B\sin px)$	1	5
	$= -p^2 y$	1	
	$\therefore \frac{d^2y}{dx^2} \propto y$	1	
VIII (1)	$y = \frac{\sin x}{\cos x}$	1	
	$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(\cos x)}{\cos^2 x}$	1	
	$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$	1	5
	$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$	1	
	$= \frac{1}{\cos^2 x} = \sec^2 x$	1	
2)	$3x^2 + 3y^2\frac{dy}{dx} = 3a\left(x\frac{dy}{dx} + y.1\right)$	1	
	$3(y^2 - ax)\frac{dy}{dx} = 3(ay - x^2)$	2	
	$\therefore \frac{dy}{dx} = \frac{3(ay - x^2)}{3(y^2 - ax)}$	1	5
	$= \frac{ay - x^2}{y^2 - ax}$	1	

VIII (3)	$y' = x^2 \cos x + 2x \sin x$	1	
	$y'' = -x^2 \sin x + 4x \cos x + 2 \sin x.$	2	
	$\therefore x^2 y'' - 4xy' + (x^2+6)y =$		
	$x^2(-x^2 \sin x + 4x \cos x + 2 \sin x) -$		5
	$4x(x^2 \cos x + 2x \sin x) +$	1	
	$(x^2+6)\, x^2 \sin x$		
	$= -x^4 \sin x + 4x^3 \cos x + 2x^2 \sin x$		
	$-4x^3 \cos x + 8x^2 \sin x + x^4 \sin x$	1	
	$+ 6x^2 \sin x \;\; \underline{\underline{= 0}}$		
IX (1)	$A = \pi r^2$		
		1	
	$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}, \quad \frac{dr}{dt} = 2$	1	
	$= 2\pi r \times 2 = 4\pi r$	1	5
	At the end of 3 second,		
	Radius, $r = 3 + 3\frac{dr}{dt} = 3+6 = 9$	1	
	When $r = 9$		
	$\frac{dA}{dt} = 4\pi \times 9 = \underline{\underline{36\pi \text{ inch}^2/\text{sec}}}$	1	
(2)	$\frac{dy}{dx} = 6x+1$		
	$\frac{dy}{dx}\Big/ (1,2) = 6+1 = 7.$	1	
	$\therefore$ Equation of the tangent at $(1,2)$ is		
	$y - y_1 = \frac{dy}{dx}(x - x_1)$	1	
	$7x - y - \underline{\underline{5 = 0}}$	1	5
	Equation of the normal at $(1,2)$ is		
	$y - y_1 = -1/\frac{dy}{dx}\,(x - x_1)$	1	
	$x + 7y - 15 \;\underline{\underline{= 0}}$	1	

IX (3)	$\frac{ds}{dt} = 6x^2 - 18x + 12$	1	
	$\frac{d^2s}{dt^2} = 12x - 18$	1	
	$\frac{ds}{dt} = 0 \Rightarrow x = 1, 2$	1	
	At $x = 1$, $\frac{d^2s}{dt^2} = -6 < 0$	1	5
	At $x = 2$, $\frac{d^2s}{dt^2} = 6 > 0$		
	$\therefore$ Deflection is maximum at $x = 1$ and the maximum deflection,		
	$S = 5$	1	
X (1)	Velocity, $v = \frac{ds}{dt} = 3t^2 - 12t + 8$	1	
	Acceleration, $a = \frac{d^2s}{dt^2} = 6t - 12$	1	5
	$a = 12$, $\therefore 6t - 12 = 12$		
	$\Rightarrow t = 4$	1	
	At $t = 4$, Velocity, $v = 8$ cm/sec	1	
(2)	Length $= 12 - 2x$ Breadth $= 12 - 2x$, Height $= x$	1	
	Volume, $V = 4x^3 - 48x^2 + 144x$	1	
	$\frac{dV}{dx} = 0 \Rightarrow 12x^2 - 96x + 144 = 0$ $\Rightarrow x = 6, 2$	1	
	$\therefore x = 2$, $x = 6$ is inadmissible		5
	At $x = 2$, $\frac{d^2V}{dx^2} = 24x - 96 = -48 < 0$	1	
	$\therefore$ Size of the square is $x = 2$.	1	

$\overline{X}$ (3)	$P = 100$ $\therefore 2(x+y) = 100 \Rightarrow x+y = 50$ $\Rightarrow y = 50 - x$. Area, $A = xy$ $= x(50-x)$. $= 50x - x^2$ $\frac{dA}{dx} = 50 - 2x$ $\frac{dA}{dx} = 0 \Rightarrow x = 25$ $\therefore y = 25$ At $x = y = 25$ $\frac{d^2A}{dx^2} = -2 < 0$ $\therefore$ Area is maximum at $\underline{\underline{x = y = 25}}$	1 1 1 1 1	5