SCHEME OF VALUATION.

Scoring Indicators.

Revision: 2015 Course Code: 2002

Course Title: Engineering Mathematics - II

Question No.	Scoring Indicator	Split up Score	Total.
I 1.	$\vec{a}+\vec{b} = 6\vec{i}+2\vec{j}-8\vec{k}$		
	$\vec{a}-\vec{b} = 4\vec{i}-4\vec{j}+2\vec{k}$	1	2
	$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 24-8-16=0$	1	
I 2.	$9x-14 = 8-4$	1	2
	$9x = 18 \Rightarrow x=2$	1	
I 3.	$\int \tan^2 x\,dx = \int(\sec^2 x-1)\,dx$	1	2
	$= \tan x - x + C$	1	
I 4.	$\left(\frac{dy}{dx}\right)^3+1 = 2\frac{dy}{dx}$		
	Order: 1	1	2
	degree: 3	1	
I 5.	$P=-4$		
	$IF = e^{\int P dx}$	1	2
	$= e^{\int -4dx} = e^{-4x}$	1	
	Part B		
II 1.	$\vec{AB} = \vec{i}+\vec{j}-2\vec{k}$, $\vec{BC} = \vec{i}-2\vec{j}+\vec{k}$		
	$\vec{AC} = 2\vec{i}-\vec{j}-\vec{k}$	2	
	$\lvert\vec{AB}\rvert = \sqrt{6}$ $\lvert\vec{BC}\rvert = \sqrt{6}$ $\lvert\vec{AC}\rvert = \sqrt{6}$	2	

$|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{AC}|$. $\therefore$ ΔABC is equilateral. 2 — 6

II 2.

$n = 9$. $\therefore$ There will be 10 terms 1

t_5 and t_6 are the middle terms. 1

$$t_5 = 9c_4 (2x)^5 \cdot \left(\frac{3}{x}\right)^4$$

$$= 326592\ x$$ 2

$$t_6 = 9c_5 (2x)^4 \left(\frac{3}{x}\right)^5$$

$$= \frac{489888}{x}$$ 2

6.

II 3.

Let $x_1 = \frac{1}{x}$ $\quad y_1 = \frac{1}{y}$ 1

$$\Delta = \begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix} = 4$$ 1

$$\Delta_1 = \begin{vmatrix} 5 & 3 \\ 3 & 5 \end{vmatrix} = 16$$ 1

$$\Delta_2 = \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix} = -4$$ 1

$$x_1 = \frac{\Delta_1}{\Delta} = 4, \quad y_1 = \frac{\Delta_2}{\Delta} = -1$$ 1

$$\therefore x = \frac{1}{4}, \quad y = -1.$$ 1

6

II 4.

$$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} \qquad 4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$ 2

$$5I = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$ 1

$$\therefore A^2 - 4A - 5I = [0]$$ 3

6.

II 5.

Let $I = \int e^x . \cos x \, dx = \int \cos x . e^x \, dx$.

$= \cos x . e^x - \int -\sin x . e^x \, dx$ 2

$= \cos x . e^x + \sin x . e^x - \int \cos x . e^x \, dx$ 2

$I + I = e^x [\cos x + \sin x] + C$ 1

$\therefore I = \frac{1}{2} e^x [\cos x + \sin x] + C$ 1

6.

II 6.

$y = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = -1, 2$ 1

$\text{Area} = \int_{-1}^{2} y \, dx$ 1

$= \int_{-1}^{2} (x^2 - x - 2) \, dx$ 1

$= \left. \frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_{-1}^{2}$ 1

$= \frac{8}{3} - 2 - 4 - \left(\frac{-1}{3} - \frac{1}{2} + 2\right)$ 1

$= -\frac{9}{2}$

$\therefore$ Area $= \frac{9}{2}$ Square units. 1

6.

II 7.

$\frac{x}{1+x^2} dx = \frac{-y}{1+y^2} dy$ 1

$\Rightarrow \int \frac{x}{1+x^2} dx = -\int \frac{y}{1+y^2} dy$ 1

Let $1+x^2 = u$

$\Rightarrow \frac{1}{2} \int \frac{1}{u} du = -\frac{1}{2} \int \frac{1}{v} dv$ $\quad 1+y^2 = v$ 1

$\Rightarrow \frac{1}{2} \log u = -\frac{1}{2} \log v + C$ 1

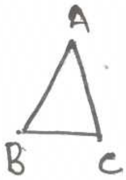
$\Rightarrow \frac{1}{2} \log(uv) = C \Rightarrow uv = e^{2C}$ 1

$\Rightarrow (1+x^2)(1+y^2) = e^{2C}$ 1

6.

Part C Unit I

III a)



$\text{Area} = \frac{1}{2} |\vec{AB} \times \vec{BC}|$ 1

$\vec{AB} \times \vec{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ -4 & -4 & 2 \end{vmatrix} = 6\vec{i} - 8\vec{j} - 4\vec{k}$ 2 5

$\therefore \text{Area} = \frac{1}{2}\sqrt{6^2 + (-8)^2 + (-4)^2} = \sqrt{29}$ Sq. units 2

III b)

$\omega = \vec{F} \cdot \vec{AB}$ 1

$\vec{AB} = 3\vec{i} + 3\vec{j} + 6\vec{k}$ 1 5

$\omega = (\vec{i} + 2\vec{j} + 5\vec{k}) \cdot (3\vec{i} + 3\vec{j} + 6\vec{k})$ 1

$= 3 + 6 + 30 = 39$ units. 2

III c)

$\left(3x + \left(\frac{-y}{2}\right)\right)^4 = (3x)^4 + 4C_1 (3x)^3 \left(\frac{-y}{2}\right) +$

$4C_2 (3x)^2 \left(\frac{-y}{2}\right)^2 + 4C_3 (3x) \left(\frac{-y}{2}\right)^3$

$+ \left(-y/2\right)^4$ 3 5

$= 81x^4 - 54x^3y + \frac{27}{2}x^2y^2 - \frac{3}{2}xy^3 + \frac{y^4}{16}$ 2

IV a)

Let $\vec{a} = 3\vec{i} + 2\vec{j} + 9\vec{k}$, $\vec{b} = \vec{i} + \mu\vec{j} + 3\vec{k}$

$\vec{a} \times \vec{b} = \vec{0}$ 1

$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 9 \\ 1 & \mu & 3 \end{vmatrix} = (6 - 9\mu)\vec{i} + 0\vec{j} +$ 3 5

$(3\mu - 2)\vec{k}$

$\vec{a} \times \vec{b} = \vec{0} \Rightarrow 6 - 9\mu = 0$ 1

$\Rightarrow \mu = 2/3$.

IV b) $\text{Proj}_{\vec{b}}\,\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ $\quad \vec{a} = \vec{i} - 2\vec{j} + 2\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} - 2\vec{k}$ — 1

$\vec{a}\cdot\vec{b} = -4 \qquad |\vec{b}| = 3$ — 2

$\text{Proj}_{\vec{b}}\,\vec{a} = \dfrac{-4}{3}$ — 2 — **5**

IV c) $t_{r+1} = 10C_r\,(3x^2)^{10-r}\left(\dfrac{-1}{2x^3}\right)^r$ — 1

$20 - 5r = 0 \Rightarrow r = 4$ — 2

$t_5 = 10C_4(3x^2)^6\left(\dfrac{-1}{2x^3}\right)^4$

$= 10C_4 \cdot \dfrac{3^6}{2^4}$ — 2 — **5**

Unit II

V a) $\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -1 & 1 \\ 2 & -7 & 3 \end{vmatrix} = 16 \qquad \Delta_2 = \begin{vmatrix} 1 & 4 & -1 \\ 3 & 4 & 1 \\ 2 & -6 & 3 \end{vmatrix} = 16$ — 1

$\Delta_1 = \begin{vmatrix} 4 & 1 & -1 \\ 4 & -1 & 1 \\ -6 & -7 & 3 \end{vmatrix} = 32 \qquad \Delta_3 = \begin{vmatrix} 1 & 1 & 4 \\ 3 & -1 & 4 \\ 2 & -7 & -6 \end{vmatrix} = -16$ — 1

$x = \dfrac{\Delta_1}{\Delta} = 2 \qquad z = \dfrac{\Delta_3}{\Delta} = -1$ — 3 — **5**

$y = \dfrac{\Delta_2}{\Delta} = 1$

V b) $AB = \begin{bmatrix} 3 & 6 & 6 \\ 6 & 9 & 12 \\ 3 & 3 & 6 \end{bmatrix} \qquad (AB)^T = \begin{bmatrix} 3 & 6 & 3 \\ 6 & 9 & 3 \\ 6 & 12 & 6 \end{bmatrix}$ — (1) — 2

$$B^T A^T = \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 3 \\ 6 & 9 & 3 \\ 6 & 12 & 6 \end{bmatrix} - (2)$$ 2 — 5

from (1) & (2) $(AB)^T = B^T A^T$ 1

V c) $\text{Adj}\,A = \begin{bmatrix} 30 & 12 & -3 \\ -20 & -8 & 2 \\ -50 & -20 & 5 \end{bmatrix}$ 3 — 5

$A \cdot \text{Adj}\,A = [0]$ 2

VI a) $-36 + 48x - 12x^2 = 0$ 3 — 5

$x^2 - 4x + 3 = 0 \Rightarrow x = 1, 3$ 2

VI b) $A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 5 & 3 & 0 \end{bmatrix}$ $A = B + C$ 1

$B = (A + A^T)\frac{1}{2} = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 2 \\ 4 & 2 & 0 \end{bmatrix}$ 2 — 5

$B^T = B$ $\therefore$ B is Symmetric

$C = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ 2

$C^T = -C$ $\therefore$ C is skew symmetric.

VI c) $A = \begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $D = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$ 1

	$A^{-1} = \frac{1}{4}\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}$	2	5
	$X = A^{-1}D = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	2	
	Unit-III		
VII a)	$\int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \cdot \sin^2 x}$	1	5
	$= \int (\operatorname{cosec}^2 x - \sec^2 x)\,dx$	2	
	$= -\cot x - \tan x + C$	2	
VII b)	$\int \tan^{-1} x \cdot 1\,dx = \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x\,dx$	2	5
	$= x\tan^{-1} x - \frac{1}{2}\int \frac{1}{u}\,du \quad u = 1+x^2$	1	
	$= x\tan^{-1} x - \frac{1}{2}\log(1+x^2) + C$	2	
VII. c)	$\int_0^{\pi/2} \sin 3x \cdot \cos x\,dx$		
	$= \frac{1}{2}\int_0^{\pi/2} (\sin 4x + \sin 2x)\,dx$	2	5
	$= \frac{1}{2}\left(-\frac{\cos 4x}{4} - \frac{\cos 2x}{2}\right)_0^{\pi/2}$	2	
	$= \frac{1}{2}$	1	
VIII a)	$\tan x = u$	1	5
	$\int (1+e^u)\,du$	1	
	$= u + e^u + C$	2	
	$= \tan x + e^{\tan x} + C$	1	

VIII b)

$$\int x^3 e^{-2x}\,dx = \frac{x^3 e^{-2x}}{-2} - \int 3x^2 \cdot \frac{e^{-2x}}{-2}\,dx \quad 2$$

$$= -\frac{x^3 e^{-2x}}{2} + \frac{3}{2}\left(-\frac{x^2 e^{-2x}}{2} - \frac{x e^{-2x}}{2} - \int \frac{e^{-2x}}{-2}\,dx\right) \quad 2$$

$$= e^{-2x}\left[-\frac{x^3}{2} - \frac{3x^2}{4} - \frac{3x}{4} - \frac{1}{4}\right] + c \quad 1$$

5

VIII c)

$$\int_0^{\pi/2} \sqrt{1+\sin 2x}\,dx = \int_0^{\pi/2} \sqrt{\sin^2 x + \cos^2 x + 2\sin x.\cos x} \quad 1$$

$$= \int_0^{\pi/2} \sin x + \cos x\,dx \quad 1$$

$$= -\cos x + \sin x \Big]_0^{\pi/2} \quad 2$$

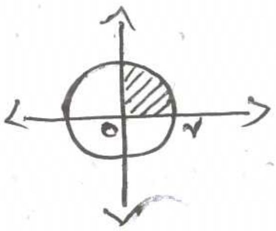
$$= 2 \quad 1$$

5

Unit - IV

IX a)

$$x^2+y^2=r^2 \Rightarrow y = \sqrt{r^2-x^2} \quad 1$$



$$\text{Area} = 4\int_0^r y\,dx \quad 1$$

$$= 4\int_0^r \sqrt{r^2-x^2}\,dx \qquad x = r\sin\theta, \quad dx = r\cos\theta\,d\theta$$

$$= 4\int_0^{\pi/2} r\cos\theta \cdot r\cos\theta\,d\theta \quad 1$$

$$= 4r^2 \int_0^{\pi/2} \frac{1+\cos 2\theta}{2}\,d\theta \quad 1$$

$$= 2r^2\left(\theta + \frac{\sin 2\theta}{2}\right)_0^{\pi/2} = \pi r^2 \quad 1$$

5

IX b)

$$\text{Volume} = \pi\int_0^a y^2\,dx \quad 1$$

$$= \frac{\pi}{a}\int_0^a ax^2 - x^3\,dx \quad 1$$

$$= \frac{\pi}{a}\left(a \cdot \frac{x^3}{3} - \frac{x^4}{4}\right)_0^a$$ 1

$$= \frac{\pi a^3}{12}$$ 2 — 5

IX c)

$$\frac{d^2y}{dx^2} = e^x + \cos x$$

$$\frac{dy}{dx} = e^x + \sin x + C_1$$ 2

$$y = e^x - \cos x + C_1 x + C_2$$ 3 — 5

X a)

$1 - 2x = x^2 - 6x + 4 \Rightarrow x = 1, 3$ 1

Required Area $= \int_1^3 [f(x) - g(x)]\,dx$ 1

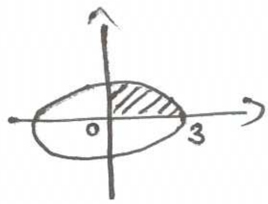
$$= \int_1^3 1 - 2x - (x^2 - 6x + 4)\,dx$$

$$= \int_1^3 (-3 + 4x - x^2)\,dx$$ 1

$$= \frac{4}{3} \text{ Square units.}$$ 2 — 5

X b)

$$y^2 = \frac{4}{9}(9 - x^2)$$ 1



Volume $V = 2\pi \int_0^3 y^2\,dx$ 1

$$= 2\pi \cdot \frac{4}{9} \int_0^3 9 - x^2\,dx$$

$$= \frac{8\pi}{9}\left(9x - \frac{x^3}{3}\right)_0^3$$ 1

$$= 16\pi \text{ Cubic units.}$$ 2 — 5

X c)

$P = \frac{3}{x}$ $\quad Q = 5x.$ 1

$$IF = e^{\int \frac{3}{x}\,dx}$$

$$= e^{3\log x} = x^3.$$ 1

Solution is $y \cdot x^3 = \int 5x \cdot x^3 \, dx$ | 2 | 5

$y x^3 = x^5 + c.$ | 1