# SCHEME OF VALUATION

Revision : 2015 Course Code : 2002

(1)

## PART A

I

1 $x^2 - 36 = 0$ — 1 $x = \pm 6$ — 1

2. $(r+1)^{th}$ term $= nC_r\, x^{n-r} a^r$ — 1

$4^{th}$ term $= 10C_3\, (x)^{10-3} \left(\frac{1}{x}\right)^3$ → 1

3 $\int (\sec^2 x - 1)\, dx = \tan x - x + c$ — 1 + 1

4. $\left[\tan^{-1} x\right]_0^1 = \pi/4$ — 1 + 1

5. $\frac{dy}{dx} = -\cos x + C_1$ 1

$y = -\sin x + C_1 x + C_2$ 1

## PART B

II

1 $\vec{AB} = 3\vec{a} - \vec{b} - 2\vec{c}$ 2

$\vec{AC} = 9\vec{a} - 3\vec{b} - 6\vec{c}$ 2

$\vec{AC} = 3\vec{AB}$ 2

2 $5^{th}$ & $6^{th}$ are the middle terms 2

$5^{th}$ term $= 9C_4\, (2x)^{9-4} \left(\frac{3}{x}\right)^4$ 2

$6^{th}$ term $= 9C_5\, (2x)^{9-5} \left(\frac{3}{x}\right)^5$ 2

3)

$$\Delta = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = -3 \quad 1$$

$$\Delta_1 = \begin{vmatrix} -3 & 2 & 1 \\ 4 & 1 & 1 \\ 6 & -1 & 2 \end{vmatrix} = -3 \quad 1$$

$$\Delta_2 = \begin{vmatrix} 1 & -3 & -1 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{vmatrix} = 3 \quad 1$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 1 & 4 \\ 1 & -1 & 6 \end{vmatrix} = -6 \quad 1$$

$x = 1 \quad y = -1 \quad z = 2 \quad$ 2

4

$$AB = \begin{bmatrix} 47 & 34 \\ 22 & 16 \end{bmatrix} \quad 1$$

$$(AB)^{-1} = \frac{\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}}{4} \quad 1$$

$$A^{-1} = \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4} \text{ — } 1 \qquad B^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \text{ — } 1$$

$$B^{-1}A^{-1} = \begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix} \Big/ 4 \longrightarrow 1$$

$$B^{-1}A^{-1} = (AB)^{-1} \text{ — } 1$$

5

$$\int_0^{\pi/2} \frac{3\sin x - \sin 3x}{4} dx = \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right]_0^{\pi/2} \longrightarrow 1 + 2$$

$$= \frac{1}{4}\left[-3\cos \pi/2 + \frac{3\cos \pi/2}{3}\right] - \frac{1}{4}\left[-3\cos 0 + \frac{\cos 0}{3}\right] \text{ — } 1$$

$$= \frac{2}{3} \longrightarrow 2$$

6. $A = \int_a^b y\,dx = \int_0^{\pi/3} \sin 3x\,dx$ —— (1+1)

$= \left[-\frac{\cos 3x}{3}\right]_0^{\pi/3} = \frac{2}{3}$ (2+2)

7. $e^{\int P dx} = e^{\log \sin x} = \sin x.$ (1)

Soln:- $y e^{\int P dx} = \int e^{\int P dx} Q\,dx$ (1)

$y \sin x = \int \sin x \cdot \operatorname{cosec} x\,dx$ (2)

$y \sin x = x + c$ (2)

PART C

Unit I.

III (a) $\vec{AB} = (3\hat{i}+\hat{j}+\hat{k}) - (\hat{i}+2\hat{j}-\hat{k})$

$= 2\hat{i}+3\hat{j}+2\hat{k}$ —— (1)

Projection of $\vec{a}$ on $\vec{b}$ $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ ⟶ (1)

$= \frac{(2\hat{i}+3\hat{j}+2\hat{k})\cdot(4\hat{i}-3\hat{j}+12\hat{k})}{\sqrt{4^2+3^2+12^2}}$ (1)

$= \frac{8-9+24}{\sqrt{169}} = \frac{23}{13}$ (1)+(1)

(b) $\vec{AB} = \hat{i}+\hat{j}+3\hat{k}$ $\vec{F} = \hat{i}+2\hat{j}+\hat{k}$ (2)

$W = \vec{F}\cdot\vec{AB} = (\hat{i}+2\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j}+3\hat{k})$ (2)

$= 6$ units

(c) $(r+1)^{th}$ term $= 9C_r (x^2)^{9-r} \left(\frac{-1}{x}\right)^r$ (1)

$= (-1)^r \, 9C_r \, x^{18-3r}$ (1)

$18 - 3r = 0 \implies r = 6$ (2)

Constant term $= 9C_6$ (1)

IV (a) Area $= \frac{1}{2} |\vec{AB} \times \vec{AC}|$ (1)

$\vec{AB} = \hat{i} - 4\hat{j} - \hat{k}$ (1)

$\vec{AC} = -2\hat{i} - \hat{j} + \hat{k}$ (1)

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & -1 \\ -2 & -1 & 1 \end{vmatrix} = -5\hat{i} + \hat{j} - 9\hat{k} \quad (1)$$

Area $= \frac{1}{2}\sqrt{107}$ (1)

(b) Moment about $A = \vec{r} \times \vec{F}$ (1)

$\vec{r} = (2\hat{i} + 3\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})$

$= \hat{i} + \hat{j} + 2k$ (1)

$\vec{F} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{r} \times \vec{F} = -3\hat{i} - \hat{j} + \hat{k}$ (2)

Moment $= \sqrt{11}$ (1)

(c) $(x-a)^n = x^n - nC_1 x^{n-1} a^1 + nC_2 x^{n-2} a^2 - nC_3 x^{n-3} a^3 + \cdots + (-a)^n$ (1)

$\left(x^3 - \frac{1}{x^2}\right)^5 = (x^3)^5 - 5C_1 (x^3)^4 \left(\frac{1}{x^2}\right)^1 + 5C_2 (x^3)^3 \left(\frac{1}{x^2}\right)^2 - 5C_3 (x^3)^2 \left(\frac{1}{x^2}\right)^3$

$+ 5C_4 (x^3) \left(\frac{1}{x^2}\right)^4 - \left(\frac{1}{x^2}\right)^5$ (2)

$= (x^3)^5 - \frac{5x^{12}}{x^2} + \frac{10x^9}{x^4} - \frac{10x^6}{x^6} + \frac{5x^3}{x^8} - \frac{1}{x^{10}}$ (1)

$= x^{15} - 5x^{10} + 10x^5 - 10 + \frac{5}{x^5} - \frac{1}{x^{10}}$ (1)

Unit II.

V.

(a)

$$2A+4B = \begin{bmatrix} 4 & 2 & 0 \\ 2 & -2 & 4 \end{bmatrix} \quad \text{—— (1)}$$

$$2A+3B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix} \quad \text{—— (2)}$$

1

(1)-(2) $\Rightarrow$ $B = \begin{bmatrix} 3 & 0 & 1 \\ 0 & -2 & 3 \end{bmatrix}$. 1+1

Substitution of $A = \begin{bmatrix} -4 & 1 & -2 \\ 1 & 3 & -4 \end{bmatrix}$. 1+1

(b)

$A^3 = A^2 \cdot A$ (1)

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & -2 \\ 6 & 0 & 1 \end{bmatrix}$. (1)

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ -15 & 1 & -3 \\ 9 & 0 & 1 \end{bmatrix}$ (1)

$$A^3 - 3A^2 + 3A - I = \begin{bmatrix} 1 & 0 & 0 \\ -15 & 1 & -3 \\ 9 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 0 & 0 \\ -21 & 3 & -6 \\ 18 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ -6 & 3 & -3 \\ 9 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (1)

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$ (1)

(c)

$$x\begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 4 & -1 \\ 2 & 3 \end{vmatrix} + 3\begin{vmatrix} 4 & 1 \\ 2 & 0 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 0 \\ -1 & 0 \end{vmatrix}$$ (2)

$3x - 20 = 7$ 2

$x = 9$ 1

VI (a)

$|A| = 3$ 1

Cofactor Matrix $= \begin{bmatrix} 1 & -2 & 1 \\ 4 & -5 & -2 \\ -2 & 4 & 1 \end{bmatrix}$ — 1

Adjoint matrix $= \begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$ — 1

$A^{-1} = \dfrac{\text{adj} A}{|A|}$ — 1

$= \dfrac{\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}}{3}$ — 1

(b) $x + 3 = 1 \Rightarrow x = -2 \rightarrow$ 1 Corresponding elements equal 1

$2y + 4 = -5 \Rightarrow y = -9/2$ — 1

$z = -8 \quad w = -8 \longrightarrow$ 1 + 1

(c) $(A + A^T)^T = A^T + A \Rightarrow A^T + A$ is Symmetric. 1

$(A - A^T)^T = -(A - A^T) \Rightarrow A - A^T$ is skewsymmetric. — 1

$\frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = \frac{1}{2}(2A) = A$ → 2 + 1

Unit III

VII

(a) (i) Put $u = \sin x \quad du = \cos x\, dx$ 1

$\int \frac{du}{\sqrt{u}} = \int u^{-1/2} du$ 1

$= \frac{u^{1/2}}{1/2} + c = \frac{\sqrt{\sin x}}{1/2} + c$ 1

(ii) $\int \sec^2(7x+2)\,dx = \frac{\tan(7x+2)}{7} + c$ — 2

(b) $\int \tan^{-1}x\,dx = \int \tan^{-1}x \cdot 1\,dx$ — 1

$= \tan^{-1}x \int 1\,dx - \int \left(\frac{d}{dx}(\tan^{-1}x) \int 1\,dx\right) dx$ — 1

$= x\tan^{-1}x - \int \frac{1}{1+x^2}x\,dx$ — 1

$x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + c$ — 2

(c) $\int_0^{\pi/2} \sin 3x \cos x\,dx = \int_0^{\pi/2} (\sin 4x + \sin 2x)\,dx$ — 1

$\frac{1}{2}\left[\frac{-\cos 4x}{2} + \frac{-\cos 2x}{2}\right]_0^{\pi/2}$ — 2

Sub $= \frac{1}{2}\left[\frac{-1}{4} + \frac{1}{2}\right] - \frac{1}{2}\left(\frac{-1}{4} + \frac{-1}{2}\right)$ — 1

$= \frac{1}{2}$ — 1

VIII) (a) $\int \frac{2+3\sin x}{\cos^2 x}\,dx = \int \frac{2}{\cos^2 x} + \frac{3\sin x}{\cos^2 x}$ — 1

$= \int 2\sec^2 x\,dx + \int 3\tan x \sec x$ — 2

$= 2\tan x + 3\sec x + c$ — 2

(b) Put $u = \sin^{-1}2x$ — 1

$du = \frac{1}{\sqrt{1-4x^2}} \cdot 2\,dx$ — 1

$\int u\frac{du}{2} = \frac{(\sin^{-1}2x)^2}{4} + c$ — 2+1

(c) $\int x^2 \log x\, dx = \int \log x \cdot x^2 dx$ (1)

$= \log x \cdot \frac{x^3}{3} - \frac{1}{3}\frac{x^3}{3}$ (2).

$\int_0^2 x^2 \log x = \frac{8}{3}\log 2 - \frac{8}{9}$ (2)

IX

## unit IV.

(a) $A = \int_a^b (f(x) - \phi(x))\, dx$ (1)

limits $a = -1$ $b = 4$ (1)

$\int_{-1}^{4} x^2 - (3x+4)\, dx$

$= \left[\frac{x^3}{3} - \frac{3x^2}{2} - 4x\right]_{-1}^{4}$ (1)

$= \frac{-125}{6}$ (2).

(b) $V = \pi \int_a^b y^2\, dx$ 1

$= \pi \int_0^2 4x\, dx = \pi\left[\frac{4x^2}{2}\right]_0^2$ (2)

$= 8\pi$ Cubi units (2)

(c) $\frac{y\, dy}{y^2+1} = \frac{x\, dx}{x^2+1}$ 1

$\int \frac{y}{y^2+1} dy = \int \frac{x}{x^2+1} dx \Rightarrow$ 1

$\frac{1}{2}\log(y^2+1) + c_1 = \frac{1}{2}\log(x^2+1) + c_2$ (3)

9

X (a) $y=0 \Rightarrow x = \pm a$

$$V = \pi \int_a^b y^2 \, dx \qquad (1)$$

$$= \pi \int_{-a}^{a} b^2 \frac{(a^2 - x^2)}{a^2} \, dx \qquad (1)$$

$$= \frac{\pi b^2}{a^2} \left[ a^2 x - \frac{x^3}{3} \right]_{-a}^{a} \qquad (1)$$

$$= \frac{\pi b^2}{a^2} \left[ a^3 - \frac{a^3}{3} - \left(-a^3 + \frac{a^3}{3}\right) \right] \qquad (1)$$

$$= \frac{4}{3} \pi a b^2 \qquad (1)$$

(b)

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1} x}}{1+x^2} \qquad (1)$$

$$\text{IF} = e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1} x} \qquad (1)$$

$$y e^{\tan^{-1} x} = \int e^{\tan^{-1} x} \cdot \frac{e^{\tan^{-1} x}}{1+x^2} \, dx \qquad (1)$$

Put $u = e^{\tan^{-1} x}$

$$y e^{\tan^{-1} x} = \frac{\left(e^{\tan^{-1} x}\right)^2}{2} + C \qquad 1+1$$

(c)

$$A = \int_a^b f(x) - g(x) \, dx \qquad 1$$

$$y = 0 \Rightarrow x^2 + x = 0 \Rightarrow x = 0 \text{ or } x = -1 \qquad (1)$$

$$A = \int_{-1}^{0} x^2 + x - 0 \, dx = \left[ \frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^{0} \qquad (1)$$

$$\left[0 + \frac{0}{2}\right] - \left[\frac{-1}{3} + \frac{1}{2}\right] = \frac{1}{6} \qquad 1+1$$