SCHEME OF VALUATION

(Scoring Indicators)

Revision: 2015 | Course code: 2002

Technical Mathematics II

Q.No	Scoring Indicator	Split up score	Sub total	Total
	Part A			
I 1.	$\sqrt{x^2+x^2+x^2}=1$, $x^2=\frac{1}{3}$, $x=\pm\frac{1}{\sqrt{3}}$	$1+\frac{1}{2}+\frac{1}{2}$	2	
2.	$\cos^2(\frac{\pi}{8})-\sin^2(\frac{\pi}{8}) = \cos(2\times\frac{\pi}{8}) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$	$1+\frac{1}{2}+\frac{1}{2}$	2	
3	$(AA^T)^T = (A^T)^T A^T = AA^T$, $\therefore AA^T$ is symmetric	1+1	2	
4.	$\int \sec x\, dx = \int \frac{\sec x(\sec x+\tan x)}{\sec x+\tan x} = \log(\sec x+\tan x)$	1+1	2	
5.	$\int\frac{dy}{y} = -3\int dx \Longrightarrow \log y = -3x+c$ OR $y = ke^{-3x}$	1+1	2	10
II 1.	Let A, B, C be the given points (a) $\vec{AB} = \hat{i}-4\hat{j}-2\hat{k}$, $\vec{AC} = -3\hat{i}+12\hat{j}+6\hat{k} = -3\vec{AB}$ $\vec{AC} \parallel \vec{AB}$, and they have a common point A $\therefore$ Points A, B, C are collinear.	1 1 1	3	
	(b) $\sin\theta = \frac{\lvert\vec{a}\times\vec{b}\rvert}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert} = \frac{35}{7\sqrt{26}} = \frac{5}{\sqrt{26}}$ $\cos\theta = \sqrt{1-\frac{25}{26}} = \frac{1}{\sqrt{26}}$ $\vec{a}\cdot\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta = \sqrt{26}\times7\times\frac{1}{\sqrt{26}} = 7$	1 1 1	3	6
2.	$T_6 = {}^{11}C_5 (3x)^{11-5}\left(\frac{5}{x}\right)^5 = {}^{11}C_5\, 3^6 . 5^5 x$	3		
	$T_7 = {}^{11}C_6 (3x)^{11-6}\left(\frac{5}{x}\right)^6 = \frac{{}^{11}C_6\, 3^5 . 5^6}{x}$	3	6	6
	(T_6, T_7 are middle terms $\longrightarrow$ Formula, $T_{r+1} = {}^nC_r\, a^{n-r} b^r \longrightarrow$	1 1)		
3.	$AB = \begin{bmatrix} -2 & 25 \\ -5 & -7 \end{bmatrix}$ $BA = \begin{bmatrix} -7 & 4 & 5 \\ -20 & 0 & 5 \\ -5 & -4 & -2 \end{bmatrix}$ $AB \neq BA$, (OR order of AB = 2 & BA = 3) $\therefore AB \neq BA$ $\therefore$ Non-commutative)	3+3	6	6

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4	(a) $4\int \frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\,dx + 7\int \frac{1}{\sin^2 x}\,dx$ $= 4\int \cot x \operatorname{cosec} x\,dx + 7\int \operatorname{cosec}^2 x\,dx$	1		
	$= -4\operatorname{cosec} x - 7\cot x + C$	1+1	3	
	(b) Put $u = 1+x^3$, $\frac{du}{3} = x^2\,dx$	1		
	$\int \frac{1}{u^4}\frac{du}{3} = \frac{1}{3}\frac{u^{-3}}{-3} = \frac{-1}{9(1+x^3)^3}$	1+1	3	6
5.	$\int_0^{\pi/2} \frac{3\cos x + \cos 3x}{4}\,dx = \left[\frac{3}{4}\sin x + \frac{\sin 3x}{4\times 3}\right]_0^{\pi/2}$	1+2		
	$= \left(\frac{3}{4}\sin\frac{\pi}{2} + \frac{\sin 3\pi/2}{12}\right) - \left(\frac{3}{4}\sin 0 + \frac{\sin 0}{12}\right)$	1+1	6	6
	$= \left(\frac{3}{4}\times 1 + \frac{(-1)}{12}\right) - 0 = \frac{8}{12} = \frac{2}{3}$.	1		
6	To find limits of integration At points of intersection of curve and the line $x^2 = -2x + 3$, $x = -3, 1$	2		
	Area $= \int_a^b [f(x) - g(x)]\,dx = \int_{-3}^{1} (3 - 2x - x^2)\,dx$	1		
	$= \left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^{1} = \left(3 - 1 - \frac{1}{3}\right) - \left(-9 - 9 + \frac{27}{3}\right)$	1+1		
	$= 2 - \frac{1}{3} + 9 = 11 - \frac{1}{3} = \frac{32}{3}$	1	6	6
7.	$\frac{dy}{dx} + \frac{1}{1+x^2}y = \frac{e^{\tan^{-1}x}}{1+x^2}$	1		
	I.F $= e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$	2		
	Solution is, $y\,e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x}}{1+x^2} e^{\tan^{-1}x}\,dx$	1		
	$= \int \frac{(e^{\tan^{-1}x})^2}{1+x^2}\,dx$ $u = \tan^{-1}x$, $du = \frac{1}{1+x^2}dx$	1		
	$= \int e^{2u}\,du = \frac{e^{2u}}{2} + C$		6	6
	$\therefore\ y\,e^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + C$ is the solution.	1		

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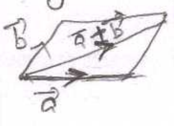
	Part.C Unit-I			
III 1.	Let $\vec{a},\vec{b},\vec{c}$ be the given vectors			
	$\lvert\vec{a}\rvert=\frac{1}{7}\sqrt{4+9+36}=\frac{7}{7}=1,\ \lvert\vec{b}\rvert=\frac{7}{7}=1\ \lvert\vec{c}\rvert=\frac{7}{7}=1$	3		
	$\vec{a}\cdot\vec{b}=\frac{1}{49}(2\times3-3\times6+6\times2)=0\Rightarrow\vec{a}\perp\vec{b}$	1		
	$\vec{b}\cdot\vec{c}=0\ ,\ \vec{c}\cdot\vec{a}=0\Rightarrow\vec{b}\perp\vec{c}\ \&\ \vec{c}\perp\vec{a}$	1+1	6	
2.	Vector product $=0\Rightarrow$ vectors are parallel			
	$\therefore\ \hat{i}+\lambda\hat{i}+\mu\hat{k}=k(2i+6\hat{j}+27\hat{k})$	2		
	$2k=1,\ 6k=\lambda\ ,\ 27k=\mu$			
	$k=\frac{1}{2}\ ,\lambda=3,\ \mu=\frac{27}{2}$	1+1	4	
	(OR by using determinants)			
3.	$T_{r+1}={}^{11}C_r(2x^2)^{11-r}\left(\frac{-3}{x}\right)^r$	1		
	$={}^{11}C_r\,2^{11-r}(-3)^r x^{22-3r}$	1		
	Let this be the term in x^{10}			
	$\therefore\ 22-3r=10\ ,\ 3r=12,\ r=4$	2		
	Coef. of $x^{10}={}^{11}C_4\,2^7(-3)^4={}^{11}C_4\,2^7\times3^4$	1	5	15
IV 1.	Resultant force, $\vec{F}=\vec{F_1}+\vec{F_2}+\vec{F_3}=3\hat{i}+4\hat{j}+5\hat{k}$	2		
	displacement vector, $\vec{d}=2\hat{i}+4\hat{j}-\hat{k}$	1		
	Total Work done $=\vec{F}\cdot\vec{d}=6+16-5=17$ units	1+1	5	
2.	diagonal vector $=\vec{a}+\vec{b}=3\hat{i}-6\hat{j}+2\hat{k}$	3		
	Its length $=7$	1		
	 Unit Vector $=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$	1	5	
3.	$T_{r+1}={}^{10}C_r(3x^2)^{10-r}\left(\frac{1}{2x^3}\right)^r={}^{10}C_r\frac{3^{10-r}}{2^r}x^{20-5r}$	2		
	$20-5r=0\ ,r=4$	2		
	Constant Term $={}^{10}C_4\times\frac{3^6}{2^4}$	1	5	15

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	Unit. II			
V 1.	$\Delta = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 10$, $\Delta_1 = \begin{vmatrix} 4 & -1 & 1 \\ 0 & 1 & -3 \\ 2 & 1 & 1 \end{vmatrix} = 20$	1+1		
	$\Delta_2 = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 0 & -3 \\ 1 & 2 & 1 \end{vmatrix} = -10$, $\Delta_3 = \begin{vmatrix} 1 & -1 & 4 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 10$	1+1		
	$x = \frac{\Delta_1}{\Delta} = 2$, $y = \frac{\Delta_2}{\Delta} = -1$, $z = \frac{\Delta_3}{\Delta} = 1$	1	5	
2.	$AB = \begin{bmatrix} 23 & 16 \\ 6 & 4 \end{bmatrix}$, $\|AB\| = -4$			
	$(AB)^{-1} = \frac{1}{-4}\begin{bmatrix} 4 & -16 \\ -6 & 23 \end{bmatrix}$	2+1		
	$\|A\| = -10+6 = -4$, $\|B\| = 21-20 = 1$			
	$A^{-1} = \frac{1}{-4}\begin{bmatrix} -2 & 3 \\ -2 & 5 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$	1+1		
	$B^{-1}A^{-1} = \frac{1}{-4}\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -2 & 5 \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} 4 & -16 \\ -6 & 23 \end{bmatrix}$	1	6	
	Hence, $(AB)^{-1} = B^{-1}A^{-1}$			
3.	$2x^2 = 0$, $-2y + x = z$, $2z - y = -\frac{5}{2}$	1		
	$x = 0$, $y = \frac{1}{2}$, $z = -1$	3	4	15
VI 1.	$3(6-18) - 1(6x - 6x^2) + 9(6x - 2x^2) = 0$	1		
	$-12x^2 + 48x - 36 = 0 \quad (\div -12)$	1		
	$x^2 - 4x + 3 = 0$, $\underline{\underline{x = 1, 3}}$	1+1	4	
2.	$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & -2 \\ 6 & 0 & 1 \end{bmatrix}$, $A^3 = \begin{bmatrix} 1 & 0 & 0 \\ -15 & 1 & -3 \\ 9 & 0 & 1 \end{bmatrix}$	2+2		
	$A^3 - 3A^2 + 3A - I = \begin{bmatrix} 1 & 0 & 0 \\ -15 & 1 & -3 \\ 9 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ -21 & 3 & -6 \\ 18 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ -6 & 3 & -3 \\ 9 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$			
	$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	2	6	

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VI. 3.	$AX = B$ where $A=\begin{bmatrix}1&1&1\\2&2&3\\1&4&9\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $B=\begin{bmatrix}1\\6\\3\end{bmatrix}$	1		
	$\lvert A\rvert = -3$	1		
	$Adj\ A = \begin{bmatrix}6&-5&1\\-15&8&-1\\6&-3&0\end{bmatrix}$, $A^{-1}=\frac{adj\ A}{\lvert A\rvert}$	1		
	$X = A^{-1}B = \frac{1}{-3}\begin{bmatrix}6&-5&1\\-15&8&-1\\6&-3&0\end{bmatrix}\begin{bmatrix}1\\6\\3\end{bmatrix} = \frac{1}{-3}\begin{bmatrix}-21\\30\\-12\end{bmatrix}$	1		
	$= \begin{bmatrix}7\\-10\\4\end{bmatrix}$	1	5	15
	$x=7,\quad y=-10,\quad z=4$			
	Unit. III			
VII 1.	$\int \tan^2 x\,dx = \int(\sec^2 x-1)\,dx = \tan x - x + C$	2+1	3	
2.	$\int\sqrt{5x+4}\,dx = \frac{1}{5}\frac{(5x+4)^{3/2}}{3/2}+C$ $= \frac{2}{15}(5x+4)^{3/2}$ (direct or by taking $u=5x+4$)	3	3	
3.	$\int_0^{\pi/4}\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$ Put $u=\tan x$, $du=\sec^2 x\,dx$	1		
	$=\int_0^1\frac{1}{\sqrt{1-u^2}}\,du = \left[\tan^{-1}u\right]_0^1 = \frac{\pi}{4}-0=\frac{\pi}{4}$			
	changing limits $\rightarrow$	1		
	Integration $\rightarrow$	1	4	
	Answer $\rightarrow$	1		
4.	$\int_0^2 x^2\log x\,dx = \left[\log x\times\frac{x^3}{3}-\int\frac{1}{x}\times\frac{x^3}{3}dx\right]_0^2$	2		
	$=\left[\frac{x^3}{3}\log x-\frac{1}{3}\int x^2dx\right]_0^2 = \left(\frac{x^3}{3}\log x-\frac{x^3}{9}\right)_0^2$	1+1		15
	$=\left(\frac{8}{3}\log 2-\frac{8}{9}\right)-0 = \frac{8}{3}\log 2-\frac{8}{9}$	1	5	

Q. No.	Scoring Indicators	Split up Score	sub-total	total
VIII 1.	$\int\left(\frac{3x+2}{\sqrt{x}}\right)dx = \int(3\sqrt{x}+2x^{-1/2})dx$	1		
	$= 3\frac{x^{3/2}}{3/2} + 2\frac{x^{1/2}}{1/2} + C = 2x^{3/2} + 4\sqrt{x} + C$	1+1	3	
2.	$\int \frac{e^{2x}}{1+e^{2x}}dx = \int \frac{du}{2u}$ Put $u = 1+e^{2x}$, $\frac{du}{2} = e^{2x}dx$	1, 1		
	$= \frac{1}{2}\log u + C = \frac{1}{2}\log(1+e^{2x}) + C$	1	3	
3.	$u = x+\cos x$			
	$du = (1-\sin x)\,dx$.	1		
	When $x=0,\ u=1$ &			
	$x=\pi,\ u=\pi+-1=\pi-1$	1		
	$\int_1^{\pi-1}\frac{du}{u^2} = \left[\frac{-1}{u}\right]_1^{\pi-1} = \left(\frac{-1}{\pi-1}\right)-\left(\frac{-1}{1}\right)$	1+1	4	
	$= \frac{-1}{\pi-1}+1.$			
4.	$\left[x\frac{\sin 3x}{3} - \int 1\times\frac{\sin 3x}{3}\right]_0^{\pi/2}$	1+1		
	$= \left[\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x\right]_0^{\pi/2}$	1		
	$= \left(\frac{1}{3}\times\frac{\pi}{2}(-1)+\frac{1}{9}\times 0\right) - \left[0+\frac{1}{9}\right]$	1+1	5	15
	$= -\frac{\pi}{6}-\frac{1}{9}$			
IX 1.	Area $= \int_0^{\pi/2}(x+\sin x)\,dx$	2		
	$= \left[\frac{x^2}{2} - \cos x\right]_0^{\pi/2}$	1+1		
	$= \left(\frac{\pi^2}{8}-0\right) - (0-1) = \frac{\pi^2}{8}+1$	1	5	
2.	When $y=0,\ x=\pm 3$, $y^2 = \frac{4}{9}(9-x^2)$	1+1		
	$V = \pi\int_a^b y^2dx = \pi\int_{-3}^{3}\frac{4}{9}(9-x^2)dx$	1		
	$= 2\pi\int_0^3\frac{4}{9}(9-x^2)dx = \frac{8\pi}{9}\left(9x-\frac{x^3}{3}\right)_0^3 = 16\pi$	1+1	5	

IX 3. $\dfrac{dy}{\sqrt{1-y^2}} = \dfrac{-dx}{\sqrt{1-x^2}}$

$\int \dfrac{dy}{\sqrt{1-y^2}} = \int \dfrac{-dx}{\sqrt{1-x^2}}$ 2+1

2

$\sin^{-1}y = -\sin^{-1}x + C.$ 5

$\therefore \sin^{-1}x + \sin^{-1}y = C$ is the solution.

X 1. When $y=0$, $x^2 - x - 2 = 0$ 2

$(x-2)(x+1)=0$, $x = 2, -1$.

Area $= \int_a^b y\,dx = \int_{-1}^{2} (x^2-x-2)\,dx$ 1

$= \left[\dfrac{x^3}{3} - \dfrac{x^2}{2} - 2x\right]_{-1}^{2} = \left(\dfrac{8}{3} - 2 - 4\right) - \left(\dfrac{-1}{3} - \dfrac{1}{2} + 2\right)$

2 5

$= -\dfrac{9}{2}$

Since area is always +ve, area $= \dfrac{9}{2}$ units

2. $y = 3\sin 2x$ meets x-axis when 2

$2x = 0$ and $2x = \pi \Longrightarrow x = 0, \pi/2$

$V = \pi \int_0^{\pi/2} y^2\,dx = \pi \int_0^{\pi/2} 9\sin^2 2x\,dx$ 1

$= 9\pi \int_0^{\pi/2} \dfrac{1-\cos 4x}{2}\,dx = \dfrac{9\pi}{2}\left[x - \dfrac{\sin 4x}{4}\right]_0^{\pi/2}$

$= \dfrac{9\pi}{2}\left(\left[\dfrac{\pi}{2} - \dfrac{\sin 2\pi}{4}\right] - \left[0 - \dfrac{\sin 0}{4}\right]\right)$ 2 5

$= \dfrac{9\pi}{2} \times \dfrac{\pi}{2} = \dfrac{9\pi^2}{4}.$

X 3.	$3e^{x}\tan y\,dx = -(1-e^{x})\sec^2 y\,dy$	1		
	$\int \frac{3e^{x}dx}{e^{2}-1} = \int \frac{\sec^2 y}{\tan y}\,dy.$	1		
	$3\log(e^{x}-1) = \log(\tan y) + C.$	2+1	5	15