Scoring Indicators

COURSE NAME: MATHEMATICS II

COURSE CODE: 2002 QID: 2106220096 A

Q No	Scoring Indicators	Split score	Sub total	Total score
	PART A			9
I.1.	$\sin^2 x + \cos^2 x = 1$	1	1	
I.2	$A+B = \begin{bmatrix} 0+3 & 0+7 \\ 2+4 & 1+8 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 6 & 9 \end{bmatrix}$	1	1	
I.3.	$3A = \begin{bmatrix} 3\times1 & 3\times4 & 3\times3 \\ 3\times2 & 3\times1 & 3\times6 \\ 3\times-1 & 3\times2 & 3\times0 \end{bmatrix} = \begin{bmatrix} 3 & 12 & 9 \\ 6 & 3 & 18 \\ -3 & 6 & 0 \end{bmatrix}$	1	1	
I.4	$\vec{PQ} = (3-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k}$ $= 2\hat{i} + 3\hat{j} + 3\hat{k}$	1	1	
I.5	$\|\vec{a}\| = \sqrt{x^2+y^2+z^2} = \sqrt{2^2+(-3)^2+1^2} = \sqrt{14}$	1	1	
I.6	$\int \sec x \tan x \, dx = \sec x + c$	1	1	
I.7	$\int e^{2x} dx = \frac{e^{2x}}{2} + c$	1	1	
I.8	Order = 3, degree = 1	1	1	
I.9	$dy = x\,dx$ $\therefore y = \frac{x^2}{2} + c$	1	1	

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II	PART B			8 x 3
II.1.	$x^2 - 36 = 16 - 3$	1		
	$x^2 = 49$	1		
	$x = \pm 7$	1	3	
II.2.	$A^T = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 1 & 2 \\ 3 & 6 & 0 \end{bmatrix}$	1		
	$A + A^T = \begin{bmatrix} 2 & 6 & 2 \\ 6 & 2 & 8 \\ 2 & 8 & 0 \end{bmatrix}$	1		
	$A - A^T = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 4 \\ -4 & -4 & 0 \end{bmatrix}$	1	3	
II.3	$\lvert A \rvert = \begin{vmatrix} 1 & 2 \\ 4 & 9 \end{vmatrix} = 9 - 8 = 1$	1		
	Adjoint of $A = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$	1		
	$A^{-1} = \frac{1}{\lvert A \rvert}$ Adjoint of $A = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$	1	3	

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II.4	$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix}$	1		
	$= (4-3)\hat{i} - (4+6)\hat{j} + (-6-12)\hat{k}$	1		
	$= \hat{i} - 10\hat{j} - 18\hat{k}$	1	3	
II.5.	$x = 2, \; y = 4, \; z = -3$	1+1+1	3	
II.6	$x^2+1 = u, \quad du = 2x\,dx$	1		
	$\therefore \int \frac{2x}{x^2+1}\,dx = \int \frac{du}{u}$			
	$= \log u + c = \log(x^2+1) + c$	1+1	3	
II.7.	$\int uv\,dx = u\int v\,dx - \int\left[\frac{d}{dx}u\left(\int v\,dx\right)\right]dx$	1		
	$\therefore \int x e^x\,dx = x\int e^x\,dx - \int \frac{d}{dx}(x)\left(\int e^x\,dx\right)dx$			
	$= xe^x - \int 1\cdot e^x\,dx$	1		
	$= xe^x - e^x + c$	1	3	
II.8	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$	1		
	$\therefore \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \left(\sin^{-1}x\right)_0^1 = \sin^{-1}(1) - \sin^{-1}(0)$	1		
	$= \frac{\pi}{2}$	1	3	

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II.9	Area $= \int_a^b y\,dx$	1		
	$= \int_1^3 (2x+3)\,dx$			
	$= \left(2\frac{x^2}{2} + 3x\right)_1^3$	1		
	$= 9 + 9 - (1+3) = 14$ sq.units	1	3	
II.10	$\frac{dx}{1+x^2} = \frac{dy}{1+y^2}$	1		
	$\therefore \int \frac{dx}{1+x^2} = \int \frac{dy}{1+y^2}$	1		
	$\therefore \tan^{-1} x = \tan^{-1} y + c$	1	3	
III	PART C			6×7
III.1	$\Delta = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = -3$	2		
	$\Delta_1 = -3, \quad \Delta_2 = 3, \quad \Delta_3 = -6$	3		
	$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}, \quad z = \frac{\Delta_3}{\Delta}$	1		
	$x = 1, \; y = -1, \; z = 2$	1	7	

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	OR			
III.2	$A = \begin{bmatrix} 5 & 2 \\ 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$	1		
	$AX = B$	1		
	$\lvert A \rvert = -5-4 = -9$	1		
	Adjoint matrix $= \begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}$			
	$A^{-1} = \frac{1}{-9}\begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}$	1		
	$\therefore X = A^{-1}B$	1		
	$= \frac{1}{-9}\begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 7 \end{bmatrix}$			
	$= \frac{1}{-9}\begin{bmatrix} -4-14 \\ -8+35 \end{bmatrix}$	1		
	$= \frac{1}{-9}\begin{bmatrix} -18 \\ 27 \end{bmatrix} = \begin{bmatrix} \frac{-18}{-9} \\ \frac{27}{-9} \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$	1	7	7
	$\therefore x = 2,\ y = -3$			

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III.3	$\vec{F} = 2\hat{i}-5\hat{j}+6\hat{k}+-\hat{i}+2\hat{j}-\hat{k}+2\hat{i}+7\hat{j}$	1		
	$= 3\hat{i}+4\hat{j}+5\hat{k}$	1		
	$\vec{AB} = 6\hat{i}+\hat{j}-3\hat{k}-(4\hat{i}-3\hat{j}-2\hat{k})$	1		
	$= 2\hat{i}+4\hat{j}-\hat{k}$	1		
	Total Workdone $= \vec{F}\cdot\vec{AB}$	1		
	$= (3\hat{i}+4\hat{j}+5\hat{k})\cdot(2\hat{i}+4\hat{j}-\hat{k})$			
	$= 6+16-5$	1		
	$= \underline{\underline{17}}$ units	1	7	
	OR			
III.4	$3\vec{a} = 3(2\hat{i}+3\hat{j}+4\hat{k}) = 6\hat{i}+9\hat{j}+12\hat{k}$	1		
	$4\vec{b} = 4(-\hat{i}+3\hat{j}+2\hat{k}) = -4\hat{i}+12\hat{j}+8\hat{k}$	1		
	$3\vec{a}+4\vec{b} = 2\hat{i}+21\hat{j}+20\hat{k}$	1		
	$\lvert 3\vec{a}+4\vec{b}\rvert = \sqrt{2^2+(21)^2+20^2} = \sqrt{845}$	2		
	unit vector in the direction of $3\vec{a}+4\vec{b}$ is $\dfrac{3\vec{a}+4\vec{b}}{\lvert 3\vec{a}+4\vec{b}\rvert}$	1		
	$= \dfrac{1}{\sqrt{845}}(2\hat{i}+21\hat{j}+20\hat{k})$	1	7	7

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III.5 (i)	$\vec{a}\cdot\vec{b} = (2\hat{i}+3\hat{j}-\hat{k})\cdot(3\hat{i}-\hat{j}+\hat{k})$ $= 6-3-1 = 2$	1+1	2	
III 5 (ii)	$\vec{F} = \hat{i}+2\hat{j}+\hat{k}$ $\vec{r} = \vec{PA} = (2\hat{i}+3\hat{j}+\hat{k})-(\hat{i}+2\hat{j}-\hat{k})$ $= \hat{i}+\hat{j}+2\hat{k}$	1		
	$\vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$	1		
	$= (1-4)\hat{i}-(1-2)\hat{j}+(2-1)\hat{k}$ $= -3\hat{i}+\hat{j}+\hat{k}$	1		
	Momemt $= \lvert\vec{r}\times\vec{F}\rvert$	1		
	$= \sqrt{9+1+1} = \sqrt{11}$	1	5	7
	OR			
III.6 (i)	Angle between $\vec{a}$ & $\vec{b}$ $= \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\,\lvert\vec{b}\rvert}\right)$	1		
	$= \cos^{-1}\left(\frac{10}{5\times 4}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$	1	2	

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III 6.(ii)	$\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}+3\hat{j}-\hat{k}$			
	$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$	1		
	$= (-1-3)\hat{i} - (-1-1)\hat{j} + (3-1)\hat{k}$			
	$= -4\hat{i}+2\hat{j}+2\hat{k}$	1		
	$\lvert\vec{a}\times\vec{b}\rvert = \sqrt{(-4)^2+2^2+2^2} = \sqrt{24}$	1		
	Unit vector perpendicular to $\vec{a}$ & $\vec{b}$ is			
	$\dfrac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert} = \dfrac{-4\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{24}}$	1+1	5	7
III.7. (i)	$\int(3x^2-4x+6)\,dx = 3\int x^2dx - 4\int x\,dx + 6\int dx$	1		
	$= 3\cdot\frac{x^3}{3} - 4\cdot\frac{x^2}{2} + 6x + C$	1		
	$= x^3 - 2x^2 + 6x + C$	1	3	
III.7 (ii)	$\int u\cdot v\,dx = u\int v\,dx - \int \frac{d(u)}{dx}\left(\int v\,dx\right)dx$	1		
	$\int x\log x\,dx = \int \log x\cdot x\,dx$	1		
	$= \log x\cdot\frac{x^2}{2} - \int \frac{1}{x}\times\frac{x^2}{2}\,dx$	1		
	$= \log x\cdot\frac{x^2}{2} - \frac{x^2}{4} + C$	1	4	7

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	OR			
III.8	(i) $\int \frac{1}{1+x^2}dx = \tan^{-1}x + c$	1		
	$\therefore \int_0^1 \frac{1}{1+x^2}dx = \left(\tan^{-1}x\right)_0^1$			
	$= \tan^{-1}1 - \tan^{-1}0$	1		
	$= \frac{\pi}{4}$	1	3	
III.8	(ii) $x^2 = u, \quad du = 2x\,dx$	1		
	$\therefore \int x \sec(x^2) \tan x^2\, dx = \int \sec u \tan u \frac{du}{2}$	1		
	$= \frac{1}{2}\sec u + c$	1		
	$= \frac{1}{2}\sec(x^2) + c$	1	4	7
III.9	$1+\sin x = u, \quad \cos x\, dx = du$	1		
	$\therefore \int \frac{\cos x}{1+\sin x}dx = \int \frac{du}{u}$			
	$= \log u + c$	1		
	$= \log(1+\sin x) + c$	1		
	$\therefore \int_0^{\pi/2} \frac{\cos x}{1+\sin x}dx = \left[\log(1+\sin x)\right]_0^{\pi/2}$	1		
	$= \log(1+\sin\frac{\pi}{2}) - \log(1+\sin 0)$	1		
	$= \log 2 - \log 1 = \log 2$	2	7	

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	OR			
III.10	(i) $\tan x = u, \quad du = \sec^2 x\, dx$	1		
	$\therefore \int e^{\tan x} \sec^2 x\, dx = \int e^u\, du$			
	$= e^u + c$	1		
	$= e^{\tan x} + c$	1	3	
III.10	(ii) $\cos^2 x = \frac{1+\cos 2x}{2}$	1		
	$\therefore \int \cos^2 x\, dx = \int \frac{1+\cos 2x}{2}\, dx$			
	$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + c$	1		
	$\therefore \int_0^{\pi} \cos^2 x\, dx = \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi}$			
	$= \frac{1}{2}\left[\pi + \frac{\sin 2\pi}{2} - \left(0 + \frac{\sin 0}{2}\right]\right.$	1		
	$= \frac{1}{2}\pi$	1	4	7
III.11	$x^2 + x = 0$	1		
	ie, $x(x+1) = 0 \Rightarrow x = 0$ or $x = -1$			
	$\therefore$ Area $= \int_a^b f(x)\, dx = \int_{-1}^{0} (x^2+x)\, dx$	1+1		
	$= \left(\frac{x^3}{3} + \frac{x^2}{2}\right)_{-1}^{0} = 0 + 0 - \left(\frac{-1}{3} + \frac{1}{2}\right)$	1+1		
	$= \frac{1}{6}$ sq.units	1	7	

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	OR			
III.12	$P = \cot x, \quad Q = 2\cos x$	1		
	$\int P\,dx = \int \cot x\,dx = \log(\sin x) + c$	1		
	$\therefore$ Integrating factor $(I.F) = e^{\int P\,dx}$	1		
	ie, $IF = e^{\log(\sin x)} = \sin x$	1		
	$\therefore$ The Solution is			
	$y.(IF) = \int Q.(IF)\,dx$	1		
	ie, $y.\sin x = \int 2\cos x.\sin x\,dx$			
	$= \int \sin 2x\,dx$	1		
	ie, $y.\sin x = -\frac{\cos 2x}{2} + c$	1	7	7