SCHEME OF VALUATION

(Scoring Indicators)

Revision: 2015 — Course Code: 1002

Course Title: Engineering Mathematics I

Qst No	Scoring Indicator	Split up Score	Sub Total	Total
I 1.	$\tan 150^\circ = \tan(180^\circ - 30^\circ)$	1	2	
	$= -\tan 30^\circ = -1/\sqrt{3}$	1		
2.	$\sec\alpha = \sqrt{1+\tan^2\alpha} = \sqrt{1+(12/5)^2} = 13/5$	1	2	
	$\cos\alpha = \frac{1}{\sec\alpha} = \frac{5}{13}$	1		
3.	Area, $\Delta = \frac{1}{2} ab \sin C$	1	2	10
	$= \frac{1}{2} \times 2 \times 4 \times \sin 30^\circ = 2\ cm^2$	1		
4.	$\lim_{x\to 3} \frac{x^3-27}{x-3} = \lim_{x\to 3} \frac{x^3-3^3}{x-3}$	1	2	
	$= 3(3)^{3-1} = 27$	1		
5.	$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$	1	2	
	slope at $x=1$ is $\left.\frac{dy}{dx}\right\|_{x=1} = \frac{3}{2}$	1		
II 1.	$\cos(A+B) = \cos A \cos B - \sin A \sin B$	1		
	$\cos(A-B) = \cos A \cos B + \sin A \sin B$	1		
	$\cos(A+B)\cos(A-B)$	1		
	$= (\cos A \cos B - \sin A \sin B)(\cos A \cos B + \sin A \sin B)$		6	6
	$= \cos^2 A \cos^2 B + \cos A \cos B \sin A \sin B$			
	$- \sin A \sin B \cos A \cos B - \sin^2 A \sin^2 B$	1		
	$= \cos^2 A \cos^2 B - \sin^2 A \sin^2 B$			
	$= \cos^2 A (1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$	1		
	$= \cos^2 A - \cos^2 A \sin^2 B - \sin^2 B + \cos^2 A \sin^2 B$	1		
	$= \cos^2 A - \sin^2 B$			

Revision: 2015 | Course Code: 1002

Course Title: Engineering Mathematics I

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2.	$\sin 120^\circ = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$	1		
	$\cos 330^\circ = \cos(360^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$	1		
	$\cos 240^\circ = \cos(180^\circ + 60^\circ) = -\cos 60^\circ = -1/2$	1	6	6
	$\sin 330^\circ = \sin(360^\circ - 30^\circ) = -\sin 30^\circ = -1/2$	1		
	$\therefore \sin 120^\circ \cos 330^\circ + \cos 240^\circ \sin 330^\circ$			
	$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + (-1/2) \times (-1/2) = 1$	2		
3.	Using cosine rule,			
	$b^2+c^2-a^2 = 2bc\cos A$, $c^2+a^2-b^2 = 2ca\cos B$,	1		
	$a^2+b^2-c^2 = 2ab\cos C$.			
	$R(a^2+b^2+c^2) = R[(b^2+c^2-a^2)+(c^2+a^2-b^2)+(a^2+b^2-c^2)]$	1		
	$= R[2bc\cos A + 2ca\cos B + 2ab\cos C]$	1		
	$= 2Rbc\cos A + 2Rca\cos B + 2Rab\cos C$		6	6
	$= \frac{a}{\sin A} bc\cos A + \frac{b}{\sin B} ca\cos B + \frac{c}{\sin C} ab\cos C$	1		
	$= abc\left[\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}\right]$	1		
	$= abc[\cot A + \cot B + \cot C]$	1		
4.	Let $a = 4k$, $b = 5k$, $c = 6k$	1		
	$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{3}{4}$	1		
	$A = \cos^{-1}(3/4) = 41^\circ 25'$	1	6	6
	$\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{9}{16}$	1		
	$B = \cos^{-1}(9/16) = 55^\circ 46'$	1		
	$C = 180^\circ - (A+B) = 82^\circ 49'$	1		

Revision: 2015 Course Code: 1002

Course Title: Engineering Mathematics I

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5.	$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$	1		
	$= \frac{\cos x \times \frac{d}{dx}(\sin x) - \sin x \times \frac{d}{dx}(-\cos x)}{\cos^2 x}$	1	6	6
	$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$	2		
	$= \frac{1}{\cos^2 x} = \sec^2 x$	2		
6.	$\frac{dy}{dx} = \frac{d}{dx}(x^2 \cos x) = 2x\cos x - x^2 \sin x$	1		
	$\frac{d^2y}{dx^2} = \frac{d}{dx}(2x\cos x - x^2 \sin^2 x)$	2	6	6
	$= (2 - x^2)\cos x - 4x \sin x$			
	$x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (x^2+6)y = 0$	3		
7.	Let x and y be the length and breadth of a rectangle of fixed perimeter l. Let A be the area of the rectangle.			
	Then $2x + 2y = l$ and $A = xy$	1		
	$\therefore y = \frac{1}{2}(l - 2x)$	1		
	$\therefore A = \frac{1}{2}lx - x^2$	1	6	6
	$\frac{dA}{dx} = \frac{1}{2}l - 2x$	1		
	$\therefore \frac{dA}{dx} = 0$ when $x = \frac{l}{4}$	1		
	$\frac{d^2A}{dx^2} = -2 < 0$	1		
	$\therefore$ Area is maximum when $x = \frac{l}{4} = y$ ie, when the rectangle is a square	1		

Revision: 2015 Course Code: 1002

Course Title: Engineering Mathematics I

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III (a)	$\sqrt{\frac{1+\sin A}{1-\sin A}} = \frac{\sqrt{1+\sin A}}{\sqrt{1-\sin A}} \times \frac{\sqrt{1+\sin A}}{\sqrt{1+\sin A}}$	2	5	
	$= \frac{1+\sin A}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$	1		
	$= \frac{1+\sin A}{\cos A} = \sec A + \tan A$	2		
(b)	$\cos^2 A = 1 - \sin^2 A = 1 - (3/5)^2 = \frac{16}{25}$	1	5	15
	$\therefore \cos A = 4/5$			
	$\tan A = \frac{\sin A}{\cos A} = \frac{3/5}{4/5} = 3/4$	1		
	$\sin^2 B = 1 - \cos^2 B = 1 - (12/13)^2 = \frac{25}{169}$	1		
	$\therefore \sin B = 5/13$			
	$\tan B = \frac{\sin B}{\cos B} = \frac{5/13}{12/13} = 5/12$	1		
	$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{56}{33}$	1		
(c)	Let $R\sin(x+\alpha) = \sqrt{3}\cos x + \sin x$	1	5	
	$R(\sin x \cos\alpha + \cos x \sin\alpha) = \sqrt{3}\cos x + \sin x$	1		
	$R\cos\alpha = 1$ and $R\sin\alpha = \sqrt{3}$			
	$R^2(\cos^2\alpha + \sin^2\alpha) = 4$	1		
	$\Rightarrow R^2 = 4 \quad \Rightarrow R = \pm 2$			
	$\frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{3}}{1}$ ie, $\tan\alpha = \sqrt{3}$	1		
	$\Rightarrow \alpha = 60^\circ$			
	$\therefore \sqrt{3}\cos x + \sin x = \pm 2\sin(x+60^\circ)$	1		
IV (a)	$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$	2	5	
	$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta} = \frac{2}{\sin\theta}$	3		
	$= 2\operatorname{cosec}\theta$			

Revision: 2015 Course Code: 1002

Course Title: Engineering Mathematics I

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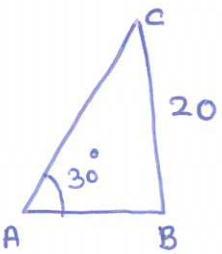
(b)	$\tan(A+B) = \tan 45^\circ = 1$	1		
	$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$	2		
	$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$		5	
	$\Rightarrow \tan A + \tan B + \tan A \tan B = 1$			
	Adding 1 on both sides,			
	$1 + \tan A + \tan B + \tan A \tan B = 1+1=2$	1		
	$\Rightarrow 1 + \tan A + \tan B(1 + \tan A) = 2$	1		
	$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$			15
(c)		1		
	Let A be the position of the boat and BC be the light house.	1	5	
	Then $BC = 20$ m			
	The distance between the boat and the observer $= AC$	1		
	$\sin 30^\circ = \frac{20}{AC}$	1		
	$\Rightarrow AC = \frac{20}{\sin 30} = \frac{20}{1/2} = 40$ m	1		
V (a)	$1 + \tan\theta \tan 2\theta = 1 + \tan\theta \times \frac{2\tan\theta}{1 - \tan^2\theta}$	2		
	$= \frac{1 - \tan^2\theta + 2\tan^2\theta}{1 - \tan^2\theta} = \frac{1 + \tan^2\theta}{1 - \tan^2\theta}$	2	5	
	$= \frac{1}{\cos 2\theta} = \sec 2\theta$	1		

Revision: 2015 Course Code: 1002

Course Title: Engineering Mathematics I

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(b)	$\cos 55^\circ + \cos 65^\circ + \cos 175^\circ$			
	$= (\cos 55^\circ + \cos 65^\circ) + \cos 175^\circ$	1		
	$= 2\cos\frac{65^\circ+55^\circ}{2}\cos\frac{65^\circ-55^\circ}{2} + \cos(180^\circ-5^\circ)$	2	5	
	$= 2\cos 60^\circ \cos 5^\circ - \cos 5^\circ$	1		
	$= 2\times\frac{1}{2}\cos 5^\circ - \cos 5^\circ = \cos 5^\circ - \cos 5^\circ = 0$	1		15
(c)	$2ab\cos C = a^2+b^2-c^2,\ 2ca\cos B = c^2+a^2-b^2$	1		
	$\therefore a(b\cos C - c\cos B) = ab\cos C - ac\cos B$	1		
	$= \frac{1}{2}[2ab\cos C - 2ac\cos B]$	1	5	
	$= \frac{1}{2}[a^2+b^2-c^2-(c^2+a^2-b^2)]$	1		
	$= \frac{1}{2}(2b^2-2c^2) = b^2-c^2$	1		
VI (a)	$\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A}$			
	$= \frac{3\sin A - 4\sin^3 A}{\sin A} - \frac{4\cos^3 A - 3\cos A}{\cos A}$	2		
	$= (3-4\sin^2 A) - (4\cos^2 A - 3)$	1	5	
	$= 6 - 4(\cos^2 A + \sin^2 A) = 6-4 = 2$	2		
(b)	$\sin 10^\circ \sin 50^\circ \sin 70^\circ$			15
	$= \frac{1}{2}\sin 10^\circ [2\sin 70^\circ \sin 50^\circ]$	1		
	$= \frac{1}{2}\sin 10^\circ [\cos(70^\circ-50^\circ) - \cos(70^\circ+50^\circ)]$	1		
	$= \frac{1}{2}\sin 10^\circ (\cos 20^\circ - \cos 120^\circ)$			
	$= \frac{1}{2}\sin 10^\circ \cos 20^\circ - \frac{1}{2}\sin 10^\circ \cos(180^\circ-60^\circ)$	1	5	
	$= \frac{1}{4}(2\cos 20^\circ \sin 10^\circ) + \frac{1}{2}\sin 10^\circ \times \frac{1}{2}$			
	$= \frac{1}{4}(\sin(20^\circ+10^\circ) - \sin(20^\circ-10^\circ)) + \frac{1}{4}\sin 10^\circ$	1		
	$= \frac{1}{4}(\sin 30^\circ - \sin 10^\circ) + \frac{1}{4}\sin 10^\circ = \frac{\sin 30^\circ}{4} = \frac{1}{8}$	1		

Revision: 2015 | Course Code: 1002

Course Title: Engineering Mathematics I

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(c)	$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2} = \frac{a-b}{a+b}\tan\left(90^\circ-\frac{C}{2}\right)$	1		
	$= \frac{34}{140}\tan 55^\circ = 0.3468$			
	$A-B = 2\tan^{-1}(0.3468) = 38^\circ 16'$	1		
	$A+B = 180^\circ - C = 110^\circ$	1	5	
	$\Rightarrow 2A = 148^\circ 16' \Rightarrow A = 74^\circ 8'$	1		
	$B = 110^\circ - A = 35^\circ 52'$			
	$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow c = \frac{a\sin C}{\sin A}$			
	$= 87\ \frac{\sin 70^\circ}{\sin 74^\circ 8'}$	1		
	$= 84.99$ cm			
VII (a)	$\lim_{x\to\infty}\frac{3x^2-5x+9}{2x^2+4x+7} = \lim_{x\to\infty}\frac{x^2\left(3-\frac{5}{x}+\frac{9}{x^2}\right)}{x^2\left(2+\frac{4}{x}+\frac{7}{x^2}\right)}$	2		
	$= \frac{3-5\lim_{x\to\infty}\frac{1}{x}+9\lim_{x\to\infty}\left(\frac{1}{x^2}\right)}{2+4\lim_{x\to\infty}\frac{1}{x}+7\lim_{x\to\infty}\left(\frac{1}{x^2}\right)}$	1	5	
	$= \frac{3-0+0}{2+0+0} = \frac{3}{2}\left(\because \lim_{x\to\infty}\frac{1}{x}=0\right)$	1+1		
(b) i)	$\frac{dy}{dx} = \frac{d}{dx}(x^2\sin^{-1}x) = x^2\frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x\frac{d}{dx}(x^2)$	1		15
	$= x^2\times\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x\times 2x$	2	3+2 = 5	
	$= x\left[\frac{x}{\sqrt{1-x^2}} + 2\sin^{-1}x\right]$			
ii)	$\frac{dy}{dx} = \frac{d}{dx}\log\sin x = \frac{1}{\sin x}\times\frac{d}{dx}(\sin x)$	1		
	$= \frac{\cos x}{\sin x} = \cot x$	1		

Revision: 2015 Course Code: 1002

Course Title: Engineering Mathematics I

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(c)	$\frac{dy}{dx} = \frac{d}{dx}(\sin x \cos x) = \sin x \times -\sin x + \cos x \cdot \cos x$			
	$= \cos^2 x - \sin^2 x = \cos 2x$	2		
	$\frac{d^2y}{dx^2} = \frac{d}{dx}(\cos 2x) = -2 \sin 2x$		5	
	$= -4 \sin x \cos x = -4y$	2		
	$\therefore \frac{d^2y}{dx^2} + 4y = 0$	1		
VIII (a) i)	$\lim_{x \to 0} \frac{2x^2+3x}{3x^2-4x} = \lim_{x \to 0} \frac{x(2x+3)}{x(3x-4)}$	1		
	$= \lim_{x \to 0} \frac{2x+3}{3x-4}$	1	3+2 = 5	
	$= \frac{-3}{4}$	1		
ii)	$\lim_{x \to 0} \frac{\sin 3x}{x} = \lim_{x \to 0} \frac{\sin 3x}{3x} \times 3$	1		
	$= 1 \times 3 = 3$	1		
(b)	Let $y = \sqrt{x}$. Then $y + \Delta y = \sqrt{x + \Delta x}$	1		15
	$\therefore \Delta y = \sqrt{x+\Delta x} - \sqrt{x}$	1		
	$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$	1	5	
	$= \lim_{x+\Delta x \to x} \frac{(x+\Delta x)^{1/2} - x^{1/2}}{(x+\Delta x) - x}$	1		
	$= \frac{1}{2} x^{(1/2)-1} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$	1		
(c)	$y' = \frac{1}{1+x^2}$, $y'' = \frac{-2x}{(1+x^2)^2}$	1+2		
	$(1+x^2)y'' + 2xy' = \frac{-2x}{1+x^2} + \frac{2x}{1+x^2} = 0$	2	5	

Revision: 2015 Course Code: 1002

Course Title: Engineering Mathematics I

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IX (a)	$\frac{dy}{dx} = \frac{-x}{\sqrt{25-x^2}}$	1		
	Slope of the tangent at $(4,3) = -\frac{4}{3}$	1		
	Equation of the tangent at $(4,3)$ is			
	$(y-3) = -\frac{4}{3}(x-4)$ ie, $4x+3y=25$	1	5	
	slope of the normal at $(4,3) = \frac{3}{4}$	1		
	Equation of the normal at $(4,3)$ is			
	$y-3 = \frac{3}{4}(x-4)$ ie, $3x-4y=0$	1		
(b)	$s = 2t^3 - 9t^2 + 12t + 6$			
	Velocity, $v = \frac{ds}{dt} = 6t^2 - 18t + 12$	2		
	Acceleration, $a = \frac{dv}{dt} = 12t - 18$	2	5	15
	Acceleration is 0 when $12t-18=0$	1		
	ie, when $t = 18/12 = 3/2$			
(c)	Let $f(x) = 4x^3 + 9x^2 - 12x + 2$			
	Then $f'(x) = 12x^2 + 18x - 12$	1		
	$= 6(2x^2 + 3x - 2)$			
	$f'(x) = 0$ when $x = 1/2$ or $x = -2$	1	5	
	$f''(x) = 24x + 18$	1		
	$f''(1/2) = 30 > 0$, $f''(-2) = -30 < 0$	1		
	$\therefore$ Maximum value $= f(-2) = 30$	1		

Revision: 2015 Course Code: 1002

Course Title: Engineering Mathematics I

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X (a)	Let $y = x^3 - 3x^2 + 6x + 7$			
	Then $\frac{dy}{dx} = 3x^2 - 6x + 6 = 3(x-1)^2 + 3 > 0$ For	3		
	all real values of x [$(x-1)^2 > 0$ For all real values of x]	1	5	
	$\therefore$ The given function is increasing For			
	all real values of x	1		
(b)	Let r be the radius and V be the			
	volume of the balloon at time t.			
	$V = \frac{4}{3}\pi r^3$	1		
	$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$	1		
	Given $\frac{dV}{dt} = 25$ cc/sec		5	
	$\therefore 25 = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{25}{4\pi r^2}$ cm/sec	1		15
	When $r = 15$ cm, $\frac{dr}{dt} = \frac{25}{4\pi \times 15^2} = \frac{1}{36\pi}$ cm/sec	2		
(c)	$\frac{dy}{dx} = 6x^2 - 18x + 12 = 6(x-1)(x-2)$	2		
	$\frac{dy}{dx} = 0$ when $x = 1$ or $x = 2$			
	$\frac{d^2y}{dx^2} = 12x - 18$	1		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 12 - 18 = -6 < 0$	1	5	
	$\left(\frac{d^2y}{dx^2}\right)_{x=2} = 12 \times 2 - 18 = 6 > 0$			
	The deflection is maximum when $x = 1$	1		
	and the maximum value $= f(1) = 5$			