COURSE: ENGINEERING MATHEMATICS-II

Revision: (2015)

Course Code: 2002

Qn. No	Scoring Indicators	Split up Score	Total
	PART A		
I 1.	$\bar{a} = 2i+3j+k$ $\quad |\bar{a}| = \sqrt{2^2+3^2+1^2} = \sqrt{14}$ —(1) Unit vector in the direction of $\bar{a} = \frac{1}{\sqrt{14}}(2i+3j+k)$ —(1)		(2)
2.	$\begin{vmatrix} x & 3 \\ 4 & 6 \end{vmatrix} = 0 \Rightarrow 6x-12=0$ —(1) $\Rightarrow 6x = 12 \Rightarrow x = 2$ —(1)		(2)
3.	$A = \begin{bmatrix} 3 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ $A+2B = \begin{bmatrix} 3 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -2 \\ 4 & 0 & 2 \end{bmatrix}$ —(1) $= \begin{bmatrix} 5 & 5 & -2 \\ 5 & -1 & 4 \end{bmatrix}$ —(1)		(2)
4.	$\int \cos^2 x \, dx = \int \frac{1+\cos 2x}{2} dx$ —(1) $= \frac{1}{2}\int (1+\cos 2x) dx.$ $= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) = \frac{x}{2} + \frac{\sin 2x}{4} + c$ —(1)		(2)
5.	$\frac{d^2y}{dx^2} = e^x$, Integrating we get $\frac{dy}{dx} = e^x + c_1$ —(1) Integrating again, $y = e^x + c_1 x + c_2.$ —(1)		(2)

Dr. MUBEEN M
Asst. Professor of Mathematics
Govt. Polytechnic College
Perinthalmanna - 679 321

PART B.

II

1. $\bar{a} = 5i - j - 3k$, $\bar{b} = i + 3j - 5k$.

$\bar{a}+\bar{b} = 6i + 2j - 8k$. —— (1)

$\bar{a}-\bar{b} = 4i - 4j + 2k$ —— (1)

To prove $\bar{a}+\bar{b} \perp^r \bar{a}-\bar{b}$, we have to s.t 6

$(\bar{a}+\bar{b})\cdot(\bar{a}-\bar{b}) = 0$. —— (2)

Now,

$(\bar{a}+\bar{b})\cdot(\bar{a}-\bar{b}) = 6\cdot 4 + 2\cdot{}^{-}4 + {}^{-}8\cdot 2$ —— (2)

$= 24 - 8 - 16 = 24 - 24 = 0$.

2. $\left(2x + \frac{3}{x}\right)^9$, $n = 9$.

Middle terms: $\frac{n+1}{2}$, $\frac{n+1}{2}+1$ terms —— (1)

i.e. $\frac{9+1}{2} = 5$, $5+1 = 6$ 2

Middle terms: 5^{th}, 6^{th} terms. —— (1)

5^{th} term: $r+1 = 5 \Rightarrow r = 4$

5^{th} term $= 9C_4 (2x)^{9-4} \left(\frac{3}{x}\right)^4$ —— (1)

$= 9C_4\, 2^5 \cdot 3^4 \cdot \frac{x^5}{x^4}$ 2 6.

5^{th} term $= 9C_4\, 2^5 \cdot 3^4 \cdot x$. —— (1)

6^{th} term: $r+1 = 6 \Rightarrow r = 5$

6^{th} term $= 9C_5 (2x)^{9-5} \left(\frac{3}{x}\right)^5$ —— (1)

$= 9C_5\, 2^4 \cdot 3^5 \cdot \frac{x^4}{x^5}$ 2

6^{th} term $= \frac{9C_5\, 2^4 \cdot 3^5}{x}$ —— (1)

Dr. MUBEEN M
Asst. Professor of Mathematics
Govt. Polytechnic College
Perinthalmanna - 679 321

3.	Cramer's rule:			
	$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -1 & 1 \\ 2 & -7 & 3 \end{vmatrix} = 16$ —— (1)	1	*	
	$\Delta_1 = \begin{vmatrix} 4 & 1 & -1 \\ 4 & -1 & 1 \\ -6 & -7 & 3 \end{vmatrix} = 32$ —— (1)	1	*	
	$\Delta_2 = \begin{vmatrix} 1 & 4 & -1 \\ 3 & 4 & 1 \\ 2 & -6 & 3 \end{vmatrix} = 16$ —— (1)	1	*	6
	$\Delta_3 = \begin{vmatrix} 1 & 1 & 4 \\ 3 & -1 & 4 \\ 2 & -7 & -6 \end{vmatrix} = -16$ —— (1)	1	*	
	$x = \frac{\Delta_1}{\Delta} = \frac{32}{16} = 2,$ $y = \frac{\Delta_2}{\Delta} = \frac{16}{16} = 1$ $z = \frac{\Delta_3}{\Delta} = \frac{-16}{16} = -1$ —— (2)	2	2	
4.	$A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$, $\lvert A \rvert = 17$ —— (1)	1		
	To find cofactor matrix.			
	$\begin{array}{cccccc} 3 & -2 & 1 & 3 & -2 & 1 \\ 2 & 1 & 0 & 2 & 1 & 0 \\ 2 & -1 & 3 & 2 & -1 & 3 \\ 3 & -2 & 1 & 3 & -2 & 1 \\ 2 & 1 & 0 & 2 & 1 & 0 \\ 2 & -1 & 3 & 2 & -1 & 3 \end{array}$			
	Cofactor Matrix, $C = \begin{bmatrix} 3 & -6 & -4 \\ 5 & 7 & -1 \\ -1 & 2 & 7 \end{bmatrix}$ —— (2)	2		

Dr. MUBEEN M
Asst. Professor of Mathematics
Govt. Polytechnic College
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$$\text{adj}(A) = C^T = \begin{bmatrix} 3 & 5 & -1 \\ -6 & 7 & 2 \\ -4 & -1 & 7 \end{bmatrix} \quad \text{——(1)}$$

1 | 6

$$A \cdot \text{adj}(A) = \begin{bmatrix} 17 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 17 \end{bmatrix} = 17 \cdot I_3 \quad \text{——(2)}$$

2

5. (i) $\int x \log x \, dx = \log x \cdot \int x \, dx - \int \frac{d}{dx}\log x \left(\int x\, dx\right) dx$ ——(1)

$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{2} \, dx$ ——(1)

$= \log x \cdot \frac{x^2}{2} - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \log x - \frac{x^2}{4} + c$ ——(1)

3 | 6

(ii) $\int \sqrt{x}(1-x)\, dx = \int x^{\frac{1}{2}}(1-x)\, dx$

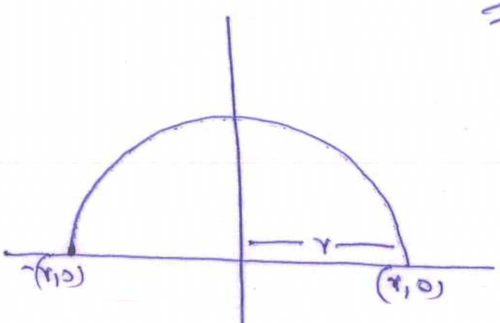
$= \int x^{\frac{1}{2}} - x^{3/2} \, dx$ ——(1)

$= \dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \dfrac{x^{3/2+1}}{\frac{3}{2}+1} + c$ ——(1)

$= \frac{2}{3} x^{3/2} - \frac{2}{5} x^{5/2} + c$ ——(1)

3

6.



——(1)

A sphere is obtained by rotating the area bounded by the semicircle about x-axis. Equation of the circle with radius 'r' centered at (0,0) is $x^2+y^2=r^2$. ——(1)

Dr. MUBEEN M
Asst. Professor of Mathematics
Govt. Polytechnic College
Perinthalmanna - 679 321

	i.e. $y^2 = r^2 - x^2$, The limits are $x=-r, x=r$.	(1)	6
	Volume $= \int_{-r}^{r} \pi y^2 dx = \pi \int_{-r}^{r} (r^2 - x^2)\, dx$.	(1)	
	$= \pi \left(r^2 x - \frac{x^3}{3}\right)_{-r}^{r}$.	(1)	
	$= \frac{4}{3} \pi r^3$	(1)	
7.	Dividing equation by (x^2+1)		
	$\Rightarrow \frac{dy}{dx} + \frac{2x}{x^2+1} \cdot y = \frac{4x^2}{x^2+1}$.		
	Which is a linear D.E where $P(x) = \frac{2x}{x^2+1}$, $Q(x) = \frac{4x^2}{x^2+1}$	(1)	
	$I.F = e^{\int P dx}$: $\int P dx = \int \frac{2x}{x^2+1} dx$.		
	put $x^2+1 = t$, $2x\,dx = dt$		
	i.e. $\int \frac{2x}{x^2+1} dx = \int \frac{dt}{t}$ $= \log(t) = \log(x^2+1)$.	(1)	6
	Integrating Factor, $I.F = e^{\int P dx} = e^{\log(x^2+1)}$ $= x^2+1$.	(1)	
	Solution is given by $y \cdot I.F = \int Q(x) \cdot I.F\, dx + C$		
	i.e. $y \cdot (x^2+1) = \int \frac{4x^2}{x^2+1} \cdot (x^2+1)\, dx + C$	(2)	
	$= \int 4x^2 dx + C$		
	$\cancel{\Rightarrow y(x^2+1) = 2x^2 + C}$		
	$= \frac{4x^3}{3} + C$		
	i.e. $y(x^2+1) = \frac{4x^3}{3} + C$ //	(1)	

Dr. MUBEEN M
Asst. Professor of Mathematics
Govt. Polytechnic College
Perinthalmanna - 679 321

Part C

UNIT - I

III (a) $\bar{a} = 2i - 3j + k$, $\bar{b} = 4i - 6j + \lambda k$.

(i) Let $\bar{a}$ and $\bar{b}$ are $\parallel^{l}$.

$\Rightarrow \frac{2}{4} = \frac{-3}{-6} = \frac{1}{\lambda} \Rightarrow \frac{1}{2} = \frac{1}{\lambda}$ — (1)

$\Rightarrow \lambda = 2$ — (1) | 2 | 5

(ii) Let $\bar{a} \perp^{r} \bar{b}$, $\Rightarrow \bar{a} \cdot \bar{b} = 0$ — (1)

$\Rightarrow 2 \cdot 4 + {}^{-}3 \cdot {}^{-}6 + 1 \cdot \lambda = 0$ — (1) | 3

$\Rightarrow 8 + 18 + \lambda = 0 \Rightarrow 26 + \lambda = 0$

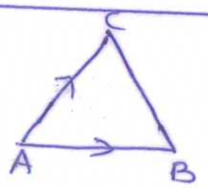
$\Rightarrow \lambda = {}^{-}26$ — (1)

(b) $\left(x^2 - \frac{1}{x^2}\right)^{20}$: $r + 1 = 10 \Rightarrow r = 9$ — (1)

10^{th} term $= 20C_9 (x^2)^{20-9} \left(\frac{-1}{x^2}\right)^9$ — (1) | 5

$= 20C_9 (x^2)^{11} \frac{(-1)^9}{(x^2)^9}$ — (1)

$= {}^{-}20C_9 \frac{x^{22}}{x^{18}} = {}^{-}20C_9 x^4$ — (2)

(c) 

Area $= \frac{1}{2} |\bar{AB} \times \bar{AC}|$. — (1)

$\bar{AB} = P \cdot V(B) - P \cdot V(A)$

$= 2i + j + 5k - (i - k)$

$= i + j + 6k$. — (1)

$\bar{AC} = P \cdot V(C) - P \cdot V(A)$

$= -i + j + 3k$. | 5.

$$\bar{AB} \times \bar{AC} = \begin{vmatrix} i & j & k \\ 1 & 1 & 6 \\ -1 & 1 & 3 \end{vmatrix} = {}^{-}3i - 9j + 2k.$$ — (2)

$|\bar{AB} \times \bar{AC}| = \sqrt{(-3)^2 + (-9)^2 + 2^2} = \sqrt{94}$

Area of $\Delta ABC = \frac{1}{2}\sqrt{94}$ — (1)

Dr. MUBEEN M
Asst. Professor of Mathematics
Govt. Polytechnic College
Perinthalmanna - 679 321

IV a) work done $= \bar{F} \cdot \overline{AB}$ — (1)

$\bar{F} = i + 2j + k$.

$\overline{AB} = p.v(B) - p.v(A)$ — (1) — 5.

$= 3i + 2j + 4k - (2i + j + k)$

$= i + j + 3k$. — (1)

$\bar{F} \cdot \overline{AB} = 1.1 + 2.1 + 1.3 = 6$ — (2)

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b) $(r+1)^{th}$ term $= 15C_r \,(x^4)^{15-r} \left(\frac{-1}{x^3}\right)^r$ — (1)

$= 15C_r (-1)^r . \dfrac{x^{60-4r}}{x^{3r}}$

$= 15C_r (-1)^r . x^{60-7r}$ — (2) — 5.

Now, $60 - 7r = 4 \Rightarrow 7r = 56$

$\Rightarrow r = 8$ — (1)

coefficient of $x^4 = 15C_8 (-1)^8$

$= 15C_8$ // — (1)

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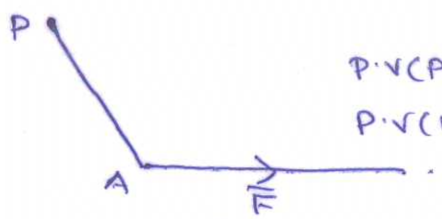
c) 

$p.v(P) = 2i + j - k$
$p.v(A) = i - j + 2k$. — (1)

Moment of force $= |\overline{PA} \times \bar{F}|$ — (1)

$\overline{PA} = p.v(A) - p.v(P)$ — 5.

$= -i - 2j + 3k$.

$\overline{PA} \times \bar{F} = \begin{vmatrix} i & j & k \\ -1 & -2 & 3 \\ 4 & 0 & 1 \end{vmatrix} = -2i + 13j + 8k$ — (2)

$|\overline{PA} \times \bar{F}| = \sqrt{237}$ // — (1).

Dr. MUBEEN M
Asst. Professor of Mathematics
Govt. Polytechnic College
[illegible]thalmanna - 679 321

V

a) $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$ — (2)

$A^2 - 5A + 6I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix}$

$+ \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ — (1)

$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$ — (2)

5.

b) $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 3 & -4 \\ 6 & 8 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 2 & 5 & 6 \\ 3 & 3 & 8 \\ 4 & -4 & 1 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 4 & 8 & 10 \\ 8 & 6 & 4 \\ 10 & 4 & 2 \end{bmatrix}$

$\frac{1}{2}(A + A^T) = \begin{bmatrix} 2 & 4 & 5 \\ 4 & 3 & 2 \\ 5 & 2 & 1 \end{bmatrix}$ which is symmetric. — 2

$(A - A^T) = \begin{bmatrix} 0 & -2 & -2 \\ 2 & 0 & -12 \\ 2 & 12 & 0 \end{bmatrix}$

$\frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -6 \\ 1 & 6 & 0 \end{bmatrix}$ which is skew symmetric. — 2

$\frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 3 & -4 \\ 6 & 8 & 1 \end{bmatrix} = A$ — (1)

5

c) $$\begin{vmatrix} 3 & 1 & 9 \\ 2x & 2 & 6 \\ x^2 & 3 & 3 \end{vmatrix} = 0 \Rightarrow -12x^2+48x-36=0 \quad (2)$$

$$\left.\begin{aligned} &\Rightarrow x^2-4x+3=0 \\ &\Rightarrow (x-1)(x-3)=0 \end{aligned}\right\} (2)$$

$$\Rightarrow x = 1, 3 \quad (1)$$

5

VI a) $$A = \begin{bmatrix} 1 & 1 & -1 \\ -2 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1+6-5 & -2+12-10 & -1+6-5 \\ 2+18-20 & 4+36-40 & 2+18-20 \\ -3-12+15 & -6-24+30 & -3-12+15 \end{bmatrix} \quad (3)$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1)$$

4

b) The given system of eqns can be written as $AX = B$ where

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

i.e. $X = A^{-1}B$ (1)

Finding A^{-1}: $|A| = 4$

$$C = \begin{bmatrix} 0 & 2 & -2 \\ -2 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix} \quad (2)$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A) = \frac{1}{4} \begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix} \quad (1)$$

$$X = \frac{1}{4} \begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (2)$$

6

$x=1, y=1, z=1.$

(c)	$\begin{vmatrix} x & 1 & 3 \\ 4 & 1 & -1 \\ 2 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 0 & 1 \\ -1 & 0 & 2 \end{vmatrix}$ $\Rightarrow \underset{(2\text{ Marks})}{3x-20} = \underset{(2\text{ Marks})}{7}$. $\Rightarrow 3x=27 \Rightarrow x=9$	(4) (1)		5
	UNIT - III			
VIII) a)	$\int \frac{4+5\cos x}{\sin^2 x}\,dx = \int \frac{4}{\sin^2 x} + \frac{5\cos x}{\sin^2 x}\,dx$. —— (1) $= \int 4\operatorname{cosec}^2 x + 5\cot x \cdot \operatorname{cosec} x\,dx$. —— (2) $= -4\cot x - 5\operatorname{cosec} x + C$ —— (2).			5.
b)	$\int \frac{1}{x(\log x)^2}\,dx$: Put $\log x = t$ —— (1) $\Rightarrow \frac{1}{x}dx = dt$ —— (1) $\Rightarrow \int \frac{1}{x(\log x)^2}\,dx = \int \frac{dt}{t^2}$ —— (1) $= \frac{-1}{t} + C$ —— (1) $= \frac{-1}{\log x} + C$ —— (1)			5.
c)	$\int \sqrt{1+\sin 2x}\,dx = \int \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}\,dx$ —— (1) $= \int \sqrt{(\sin x + \cos x)^2}\,dx$ $= \int (\sin x + \cos x)\,dx$. —— (1) $= -\cos x + \sin x \cancel{+C}$ —— (1) Hence $\int_0^{\pi/2} \sqrt{1+\sin 2x}\,dx = \left(-\cos x + \sin x\right)_0^{\pi/2}$ $= -(\cos\pi/2 + \sin\pi/2) - (-\cos 0 + \sin 0)$ —— (1)			5.

$= 0+1-(-1+0) = 2$ —— (1)

VIII

a) $\int \frac{2x^4}{1+x^{10}}\,dx$: put $x^5 = t$ — (1)

$\Rightarrow 5x^4\,dx = dt$

$\Rightarrow x^4\,dx = \frac{dt}{5}$ — (1)

5.

$\int \frac{2x^4}{1+x^{10}}\,dx = \frac{2}{5}\int \frac{dt}{1+t^2}$ — (1)

$= \frac{2}{5}\tan^{-1}(t) + C$ — (1)

$= \frac{2}{5}\tan^{-1}(x^5) + C.$ — (1)

b) $\int \tan^{-1}x\,dx = \int \tan^{-1}x \cdot 1\,dx.$

$= \tan^{-1}x \cdot \int 1\,dx - \int \frac{d}{dx}\tan^{-1}x\left(\int 1\,dx\right)dx.$ — (1)

$= \tan^{-1}x \cdot x - \int \frac{1}{1+x^2} \cdot x\,dx.$ — (2)

5

$= \tan^{-1}x \cdot x - \frac{1}{2}\log(1+x^2) + C$ — (2)

(By putting $t = 1+x^2$)

c) $\int_0^{\sqrt{\pi/2}} x\sin(x^2)\,dx$: put $x^2 = t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = dt/2.$ — (1)

when $x = 0$, $t = 0^2 = 0$

$x = \sqrt{\pi/2}$, $t = \left(\sqrt{\frac{\pi}{2}}\right)^2 = \pi/2.$ — (1)

5.

i.e. $\int_0^{\sqrt{\pi/2}} x\sin(x^2)\,dx = \int_0^{\pi/2} \sin t \cdot \frac{dt}{2}$ — (1)

$= \frac{1}{2}\int_0^{\pi/2} \sin t\,dt$

$= \frac{1}{2}\left(-\cos t\right)_0^{\pi/2} = \frac{1}{2}\left(\cos t\right)_{\pi/2}^{0}$ — (1)

$= \frac{1}{2}(1-0) = \frac{1}{2}$ — (1)

Dr. MUBEEN M
Asst. Professor of Mathematics
Govt. Polytechnic College
Perinthalmanna - 679 321

UNIT IV

IX a)	At the points where the curve meets the X-axis, $y=0$ i.e. $2\sin 3x = 0 \therefore x = 0, \pi/3$ — (1) Area $= \int_a^b y\,dx = \int_0^{\pi/3} 2\sin 3x\,dx$ — (1) $= \frac{-2}{3}\left(\cos 3x\right)_0^{\pi/3}$ — (1) $= \frac{-2}{3}\left(\cos\pi - \cos 0\right)$ $= \frac{4}{3}$ sq. units. — (2)			5
b)	$y = 2x^2+1$, i.e. $x^2 = \frac{y-1}{2}$ — 1 Volume $= \pi\int_a^b x^2\,dy$ $= \pi\int_8^9 \left(\frac{y-1}{2}\right) dy$ — (1) $= \frac{\pi}{2}\left(\frac{y^2}{2} - y\right)_8^9$ — (1) $= \frac{\pi}{2}\left[\left(\frac{81}{2}-9\right) - \left(\frac{64}{2}-8\right)\right]$ $= \frac{\pi}{2}\left[\frac{63}{2} - 24\right]$ $= \frac{\pi}{2}\cdot\left(\frac{63-48}{2}\right) = \frac{\pi}{2}\cdot\frac{15}{2}$ $= \frac{15\pi}{4}$. — (2)			5
c)	This is a linear D.E with $P = \cot x$, $Q = \operatorname{cosec} x$ — (1) $\int P\,dx = \int \cot x = \log \sin x$ — (1) $I.F = e^{\int P\,dx} = e^{\log \sin x} = \sin x$. — (1) Solution: $y\, I.F = \int Q \cdot I.F\,dx + C$ — (1) i.e. $y\cdot\sin x = \int \operatorname{cosec} x \cdot \sin x\,dx + C$			5.

$y\sin x = x + C$ — (1)

X a) $y^2 = x$ —— ① $\quad x^2 = y$ —— ②

solving ① and ②, $x^4 = x \Rightarrow x = 0, 1$ —— (1)

Area $= \int_a^b y_1 - y_2 \, dx$ —— (1)

$= \int_0^1 (\sqrt{x} - x^2) \, dx.$

$= \left(\frac{2}{3} x^{2/3} - \frac{x^3}{3}\right)_0^1$ —— (2)

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ sq. units —— (1)

5

b) $\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$: which is Variable seperable form. —— (1)

Integrating $\int \frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = c$ —— (1)

$\Rightarrow \sin^{-1}(y) + \sin^{-1}(x) = c$ —— (2)

5

c) $x\frac{dy}{dx} - 2y = x^4 - x^2$, Dividing by x,

$\Rightarrow \frac{dy}{dx} - \frac{2}{x} y = x^3 - x.$

which is a linear D.E with $P = \frac{-2}{x}$ and $Q = x^3 - x$ —— (1)

$\int P\,dx = \int \frac{-2}{x} dx = -2\log x = \log x^{-2}$ ~~$= \log\frac{1}{x^2}$ —— (1)~~

$= \log \frac{1}{x^2}$ —— (1)

I.F $= e^{\int P dx} = e^{\log \frac{1}{x^2}} = \frac{1}{x^2}$ —— (1)

Solution: $y.(I.F) = \int Q.(I.F)\,dx + C$ —— (1)

i.e. $y.\frac{1}{x^2} = \int (x^3 - x).\frac{1}{x^2}\,dx + C$

$= \int \left(x - \frac{1}{x}\right)dx + c$

$\frac{y}{x^2} = \frac{x^2}{2} - \log x + c$ —— (1)

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Dr. MUBEEN M
Asst. Professor of Mathematics
Govt. Polytechnic College
Perinthalmanna - 679 321