SCHEME OF VALUATION

(Scoring Indicators)

Revision: 2015 Course Title: **CONTROL SYSTEM**		Course code: **5043**		
Qst No	Scoring Indicator	Split Up Score	Sub Total	Total

Part A

1.	No. of poles at origin of the open loop transfer function G(s)H(s).	2		
2.	$\mathcal{L}u(t) = \frac{1}{s}$	2		
3.	Ratio of Laplace Transform of o/p signal to Laplace transform of i/p signal with all initial conditions are zero.	2		
4.	Gain Margin: The value by which the gain of the sm has to be increased to drive sm to be verge of instability. $Kg = \frac{1}{\lvert G(j\omega) \rvert_{\omega=\omega_{pc}}}$ and $Kg \text{ in db} = 20\log \frac{1}{\lvert G(j\omega)\rvert_{\omega=\omega_{pc}}} = -20\log \lvert G(j\omega)\rvert_{\omega=\omega_{pc}}$.	2		
5.	The points where ~~to~~ two root locus branches meet on the real axis and continue on this axis as k increases are known as the break in points.	2		

Part B

11) 1.

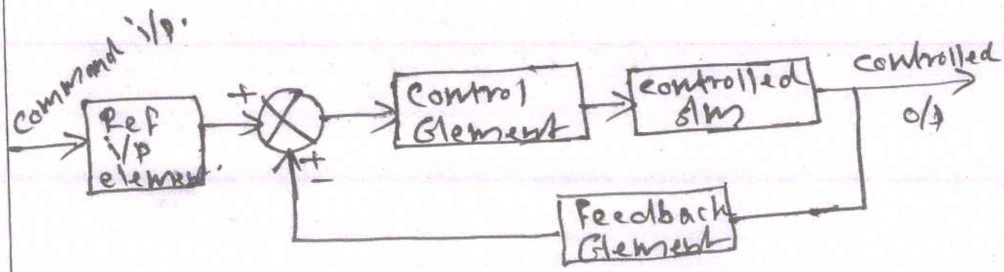


fig 1

Expl. 2

2. Gain cross over frequency: Frequency at which magnitude of the open loop transfer function is unity. 3

Phase cross over frequency: Frequency at which the phase of the open loop transfer function is 180° 3

3. The kp, kv and ka are called static error constants.

The positional error constant,

$k_p = \lim_{s\to 0} G(s)H(s)$. The steady state error in type 0 system when the i/p is unit step is given by $\frac{1}{1+k_p}$ 3

The velocity error constant, $k_v = \lim_{s\to 0} sG(s)H(s)$ 3

The steady state error in type 1 system when the i/p is unit ramp i/p is given by $\frac{1}{k_v}$

4. Absolute stability means whether system is stable or unstable 3

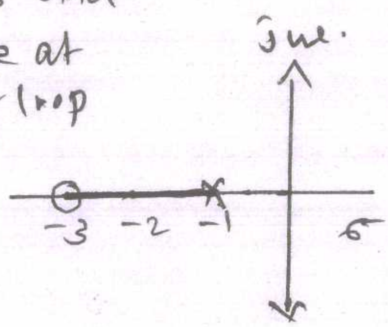
Relative stability gives the degree of stability or how close it is to instability. 3

5 $G(s) = (s+3)/(s+1)$ 1

Open loop pole is at $s=-1$ 1

Open loop zero is at $s=-3$

Here only one root locus and starts from open loop pole at $s=-1$, terminates at open loop zero at $s=-4$ 2



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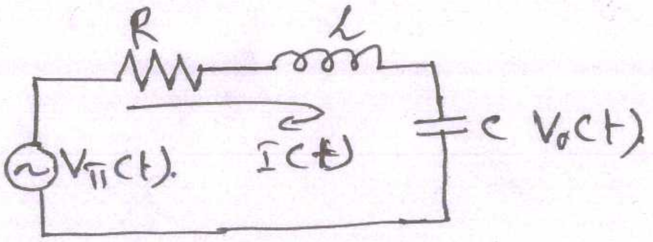
6.



Input Variable $= V_{in}(t)$.

o/p Variable $= V_o(t)$.

Equations.

$$V_{in}(t) = R\,I(t) + L\frac{d\,I(t)}{dt} + \frac{1}{C}\int I(t)\,dt$$ 2

$$V_o(t) = \frac{1}{C}\int I(t)\,dt$$

Taking Laplace Transforms

$$V_{in}(s) = R\,I(s) + sL\,I(s) + \frac{1}{sC}I(s)$$ 2

$$V_o(s) = \frac{1}{sC}I(s)$$

$$\text{Transfer fn} = G(s) = \frac{V_o(s)}{V_{in}(s)}$$

$$= \frac{\frac{1}{sC}I(s)}{R\,I(s) + sL\,I(s) + \frac{1}{sC}I(s)}$$ 2

$$G(s) = \frac{1}{s^2LC + sRC + 1}$$

7.	$L.T(e^{-at}) = \int_0^{\infty} e^{-at} e^{-st}\, dt = \dfrac{1}{s+a}$	3		
	$L.T(\cos at) = \dfrac{s}{s^2+a^2}$	3		
III	PART C.			
	a. ~~If L.T. of f(t) is F(s) then~~			
	$\cancel{L \frac{d\, f(t)}{d(t)} = [sF(s) - f(0^+)]}$			
	$\cancel{L \frac{d^2 f(t)}{dt^2} = [s^2F(s) - sf(0^+) - f'(0^+)]}$			
	$\cancel{L \frac{d^3 f(t)}{dt^3} = [s^3F(s) - s^2f(0^+) + sf'(0^+) - f''(0^+)]}$			
	$L \int f(t)\,dt = \dfrac{F(s)}{s} + f^{-1}(0)$			
	$L \iint f(t)\,dt = \dfrac{F(s)}{s^2} + \dfrac{f^{-1}(0)}{s^2} + \dfrac{f^{-1}(0)}{s}$	5		
	Proof			
	Let $f(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$			
	$\frac{d}{dt} f(t) = x(t)$			
	$f(0^-) = \int_{-\infty}^{0} x(\tau)\,d\tau$			
	$\therefore L\left[\frac{d}{dt} f(t)\right] = L[x(t)]$			
	$sF(s) - f(0^-) = X(s)$			
	$F(s) = \dfrac{X(s)}{s} + \dfrac{f(0^-)}{s}$			
	$= \dfrac{X(s)}{s} + \int_{-\infty}^{0} \dfrac{x(\tau)\,d\tau}{s}$			
	$L\left[\int_{-\infty}^{t} x(\tau)\,d\tau\right] = \dfrac{X(s)}{s} + \dfrac{1}{s}\int_{-\infty}^{0} x(\tau)\,d\tau.$			

b. $\dfrac{2s^2-4}{(s+1)(s-2)(s-3)}$

Let $\dfrac{2s^2-4}{(s+1)(s-2)(s-3)} = \dfrac{A}{(s+1)} + \dfrac{B}{(s-2)} + \dfrac{C}{(s-3)}$ 3

$2s^2-4 = A(s-2)(s-3) + B(s+1)(s-3) + C(s+1)(s-2).$

put $s=-1$, $A = -1/6$ 10

$s = 2$ $B = -4/3$

$s = 3$ $C = 7/2$ 3

$\dfrac{2s^2-4}{(s+1)(s-2)(s-3)} = \dfrac{-1}{6(s+1)} - \dfrac{4}{3(s-2)} + \dfrac{7}{2(s-3)}$

$L^{-1}\dfrac{2s^2-4}{(s+1)(s-2)(s-3)} = \dfrac{-1}{6}e^{-t} - \dfrac{4}{3}e^{2t} + \dfrac{7}{2}e^{3t}.$ 4

IV

a. $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 12$

$s^2Y(s) + - sy(0) - \dot{y}(0) + 5sY(s) - y(0)$ 2

$+ 6Y(s) = \dfrac{12}{s}$

$s^2Y(s) + 5sY(s) + 6Y(s) = \dfrac{12}{s}$

$Y(s) = \dfrac{12}{s(s^2+5s+6)} = \dfrac{12}{s(s+2)(s+3)}$ 2

$= \dfrac{A}{s} + \dfrac{B}{s+2} + \dfrac{C}{s+3}$

$A = s \cdot Y(s)\big|_{s=0} = 2$ 4

$B = (s+2)Y(s)\big|_{s=-2} = -6$

$C = (s+3)Y(s)\big|_{s=-3} = 4$

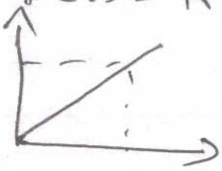
$\therefore \quad Y(s) = \frac{2}{s} + \frac{-6}{s+2} + \frac{4}{s+3}$

$y(t) = \underline{\underline{2 - 6e^{-2t} + 4e^{-3t}}}$ 2 10

b. Ramp input (velocity fn)

$f(t) = R(t)$. $\quad f(t) = 0$ for $t < 0$

$f(t) = Rt$ for $t > 0$



$F(s) = \int_0^\infty R(t)\, e^{-st}\, dt$.

$F(s) = Rt\left[\frac{e^{-st}}{-s}\right]_0^\infty - \int_0^\infty R \cdot 1 \cdot \frac{e^{-st}}{-s}\, dt$ 5

$= \frac{R}{s}\int_0^\infty e^{-st}\, dt = \frac{R}{s}\left[\frac{e^{-st}}{-s}\right]_0^\infty = \frac{-R}{s^2}[0-1]$

$= \underline{\underline{\frac{R}{s^2}}}$

V $L_1 = G_4(-H_1)$ $L_2 = G_2G_4(-H_2)$ $L_3 = G_2G_5(-H_2)$

$\Delta = 1 + G_4H_1 + G_2G_4H_2 + G_2G_5H_2$

$P_1 = G_1G_2G_3 \cdot 1$ $\quad P_2 = G_1G_2G_5 \cdot 1$ $\quad P_3 = G_3$

$L_1 = -G_4H_1$ not touching path P_3. 15

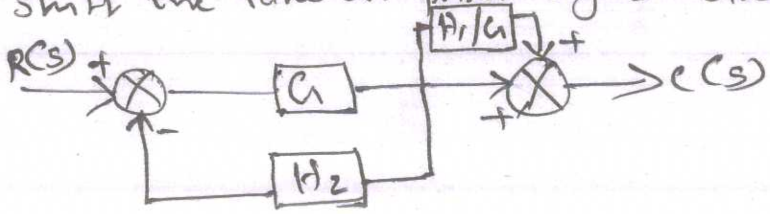
$\Delta_1 = 1$ $\quad \Delta_2 = 1$

$\Delta_3 = (1 + L_1) = [1 + (-G_4H_1)] = 1 + G_4H_1$

$$\frac{C}{R} = \frac{G_1G_2G_4 + G_1G_2G_5 + G_3(1 + G_4H_1)}{1 + G_4H_1 + G_2G_4H_2 + G_2G_5H_2}$$

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

VI (a) shift the take off point beyond block G.



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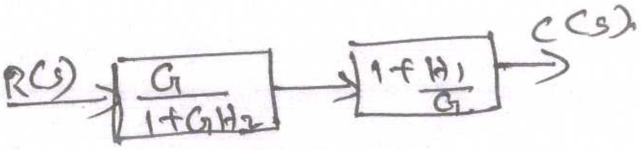
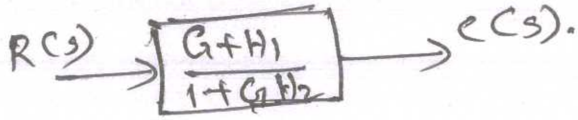




(b) Systems that can be represented by the same differential model but that are different physically are called analogous system.

6

Mechanical S/m	Electrical S/m.
Force F	Voltage e
Velocity v	current I
Displacement x	charge q
Frictional Co-efficient B	Resistance R
Mass M	Inductance L
Stiffness of spring k	Inverse of capacitance $1/C$

VII (a) $s^6 + s^5 + 5s^4 + 3s^3 + 2s^2 - 4s - 8 = 0.$

s^6	1	5	2	-8
s^5	1	3	-4	0
s^4	2	6	-8	0
s^3	0	0	0	0
s^2				

$0 \Rightarrow$ Aux equation is

$A(s) = (2s^4 + 6s^2 - 8)$.

$$\frac{dA(s)}{ds} = 8s^3 + 12s - 0$$

s^6	1	5	2	-8
s^5	1	3	-4	0
s^4	2	6	-8	0
s^3	8	12	0	0
s^2	3	-8	0	0
s^1	$\frac{100}{3}$	0	0	0
s^0	-8	0	0	0

10

There is one sign change in the first column. System is unstable.

(b) $r(t) = t$, $R(s) = \frac{1}{s^2}$

$$C(s) = \frac{1}{1+sT} R(s).$$

5

put $R(s) = \frac{1}{s^2}$

$$C(s) = \frac{1}{s^2(1+sT)}$$

By partial fraction

$$C(s) = \frac{1}{s^2(1+sT)} = \frac{As+B}{s^2} + \frac{C}{sT+1}$$

$A = -T$, $B = 1$, $C = T^2$

$$\therefore C(s) = \frac{-Ts+1}{s^2} + \frac{T}{sT+1} \qquad c(t) =$$ 5

$$\mathcal{L}^{-1}\left[\frac{-Ts+1}{s^2} + \frac{T}{sT+1}\right] = -T + t + Te^{-t/T}.$$

VIII

(a) Routh stability criteria provides information on the absolute stability of a linear time invariant system with constant co-efficients.

For stability of a closed loop system no roots of characteristic equation should be on the RHS of s-plane. 10

Procedure is as follows

① Write the characteristic equation in the following form

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n = 0$$

The necessary but not sufficient condition to be satisfied for all the roots in the RHS of s plane is

① All the co-efficients should be of the same sign

② None of the co-efficient should vanish.

If any of the co-efficients are zero or -ve, there is root or roots which are imaginary or which has positive real parts. For such a case the system is not stable and for absolute stability.

③ If all co-efficients are +ve, arrange the co-efficients of the polynomial in rows and columns according to following.

s^n : a_0 a_2 a_4 a_6 - - -

s^{n-1} : a_1 a_3 a_5 a_7 - - - -

s^{n-2} : b_0 b_1 b_2 b_3

s^{n-3} : c_0 c_1 c_2

⋮

s^1 : g_0

s^0 : h_0

$$b_0 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_1 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$c_0 = \frac{b_0 a_3 - a_1 b_1}{b_0}$$

This row is continued until the nth row has been completed.

The Routh criterion states that the system represented by the array is stable if there is no sign change in the first column of the array. The number of sign changes equals the number of roots with positive real parts.

(b) $G(s)H(s) = \dfrac{k(1+sT_a)(1+sT_b)\cdots}{(1+sT_1)(1+sT_2)\cdots}$

$K_p = \lim_{s \to 0} G(s)H(s)$

$= \lim_{s \to 0} \dfrac{k(1+sT_a)(1+sT_b)}{(1+sT_1)(1+sT_2)\cdots}$ 5

$\therefore K_p = k$

$e_{ss} = \dfrac{1}{1+K_p} = \dfrac{1}{1+k}.$

IX (a) Step1: Locate the poles and zeros of G(s)H(s) on the s plane

Step2: Determine root locus on real axis.

Step3: Determine Asymptotes of root locus and meeting pt with real axis. 10.

Step4: Find break away and break in pts

Step5: Find the angle of departure and angle of arrival

Step6: Find the crossing pt of root locus crosses the imaginary axis

Step7: sketch the root locus using the test pt by smooth curve.

(b) (1) The magnitudes are expressed in db and so a simple procedure is available to add magnitude of each term one by one. 5

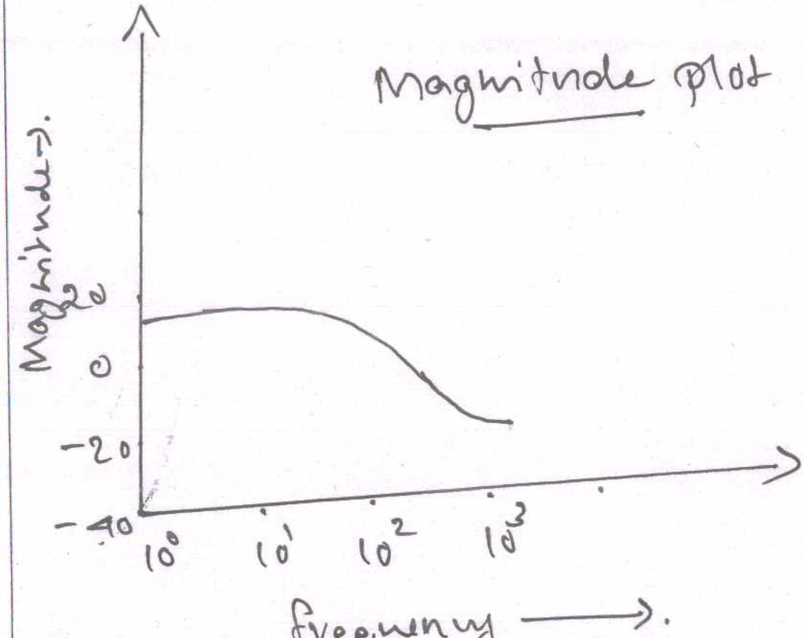
(2) The approximate Bode plot can be quickly sketched; and the corrections can be made at corner frequencies to get the exact plot.

(3) The frequency domain specifications can be easily determined. 5

(4) The Bode plot can be used to analyse both open loop and closed loop system.

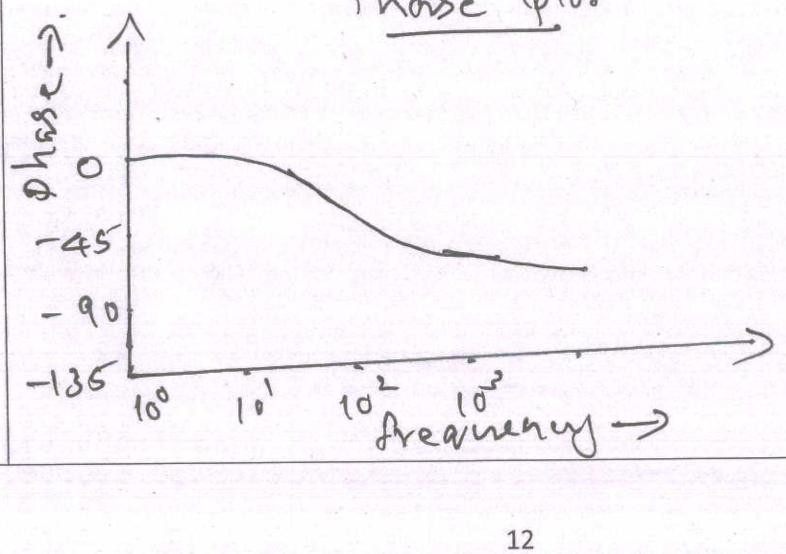
X(a)

Magnitude plot



5

Phase plot



4

(b) Phase Margin

Amount of additional phase lag at gain cross over frequency (ω_{gc}) required to bring the sm to the verge of instability. 6

It is $180^\circ + \phi_{gc}$.

where ϕ_{gc} is the phase angle at gain cross over frequency.

Gain cross over frequency is the frequency at which magnitude of the open loop transfer function is unity.