8/11/23 (32) (10) NOV-23

SCORING INDICATORS

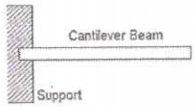
COURSE NAME: REV (21) 3021 STRENGTH OF MATERIALS

COURSE CODE: 3021 QID: 2110220251

Q No	Scoring Indicators	Split Score	Sub Total	Total Score
	PART A			
1	Within elastic limit, Stress is directly proportional strain			1
2	Ratio of Lateral strain to longitudinal strain under an axial load			1
3	Overhanging beams projects beyond one or both ends of the beam, also it may carry loads outside the support			1
4	The rate of change of bending moment with respect to the length of the beam is equal to the shear force			1
5	Load W = w(N/m) x L (length of the UDL)			1
6	Neutral layer is one in which there is no strain or stress			1
7	Slenderness ratio classify the columns into short, medium and long column			1
8	It is the load per unit deflection			1
9	Hoop stress is related to diameter and thickness of the thin cylinder, large stress cause fracture of the cylinder into two halves			1
	PART B			
1	Area, $A = 0.02 \times 0.02 = 0.0004\ m^2$ Length, $l = 50$ mm or 0.05 m Load, $P = 100$ kN $E = 2.14 \times 10^8\ kN/m^2$ **Shortening of the rod δl:** Stress, $\sigma = \frac{P}{A}$ $\therefore \sigma = \frac{100}{0.0004} = 250000\ kN/m^2$ Also, $E = \frac{\text{Stress}}{\text{Strain}}$ or, $\text{Strain} = \frac{\text{Stress}}{E} = \frac{250000}{2.14 \times 10^8}$ or, $\frac{\delta l}{l} = \frac{250000}{2.14 \times 10^8}$ $\therefore \delta l = \frac{250000}{2.14 \times 10^8} \times l = \frac{250000}{2.14 \times 10^8} \times 0.05$ $= 0.0000584$ m or 0.0584 mm *Hence, the shortening of the rod* = **0.0584 mm (Ans.)**			3
2	**(*iv*) Percentage elongation :** Percentage elongation $= \frac{\text{Length of specimen at fracture} - \text{original length}}{\text{Original length}}$ $= \frac{249 - 200}{200} = 0.245 =$ **24.5% (Ans.)**			3
3	• A **beam** is a structural element that primarily resists loads applied laterally (perpendicular) to the **beam's** axis. • Its mode of deflection is primarily by bending. • Depending upon the end condition, Beams are classified as 1. ***Cantilever beam*** 2. ***Simply Supported beam***			3

3. ***Overhanging beam***
4. ***Continuous beam***
5. ***Fixed beam***

Types of Beams

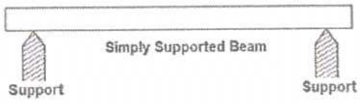


Cantilever Beam:

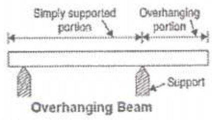
- This beam is fixed rigidly at one end and free at the other end
- There is a vertical reaction and moment at fixed end

Simply Supported Beam:

- This beam is supported at its two ends
- Two Support can be both roller or one roller and one hinge type
- There is a vertical reactions on both ends (*for hinge support – there will be horizontal reaction)*



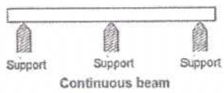
Types of Beams



Overhanging beam

- In this beam the end portion of a beam is extended beyond the support
- There are reactions at the support

Continuous Beam



- This beam is provided with more than 2 supports
- There are reactions at the support

Fixed Beam (Built in beam)

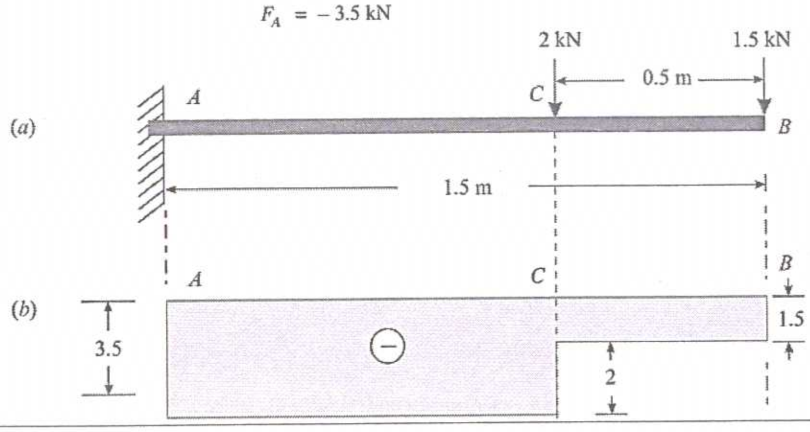
- This beam fixed rigidly at both ends
- There are reactions and resisting moments at fixed ends

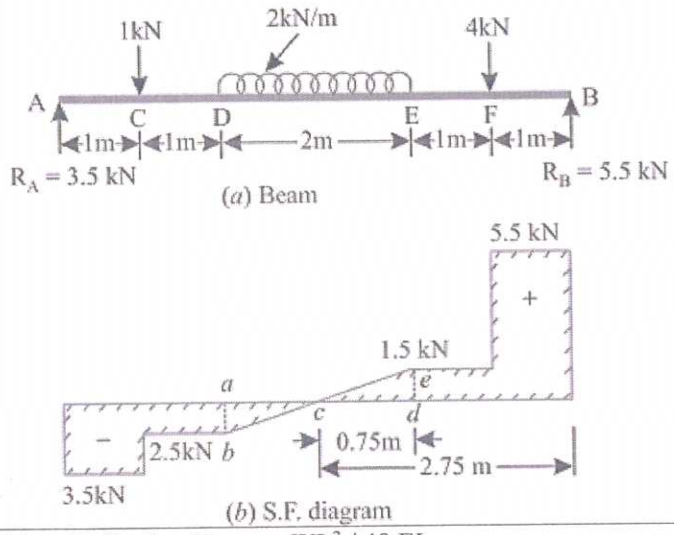
4 SOLUTION. Given : Span (l) = 1.5 m ; Point load at B (W_1) = 1.5 kN and point load at C (W_2) = 2 kN. 3

Shear force diagram

The shear force diagram is shown in Fig. 13.3 (*b*) and the values are tabulated here:

$$F_B = -W_1 = -1.5 \text{ kN}$$
$$F_C = -(1.5 + W_2) = -(1.5 + 2) = -3.5 \text{ kN}$$
$$F_A = -3.5 \text{ kN}$$



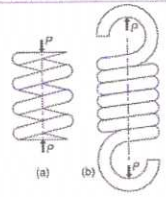
<table>
<tr><td>5</td><td>
</td><td></td><td></td><td>3</td></tr>
<tr><td>6</td><td>Max. Deflection Ymax = $WL^3 / 48\ EI$
W = 2500 N</td><td></td><td></td><td>3</td></tr>
<tr><td>7</td><td>$P = \Pi^2\ E\ I / L^2 \ \ = 9.46 \times 10^7$ N</td><td></td><td></td><td>3</td></tr>
<tr><td>8</td><td>

Helical Springs

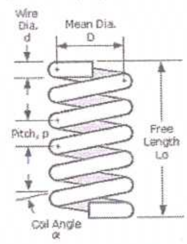
- The helical springs are made up of a *wire coiled in the form of a helix* and is intended for ***compressive or tensile loads.***
- The cross-section of the wire from which the spring is made may be ***circular, square or rectangular.***

The two forms of helical springs are

- ***Open- Coil helical Spring***
- Helical spring is said to be ***open coiled*** if the wire is coiled in such a way that *there is a gap between the two consecutive turns ;* ***helix angle (α) is large > 10º***
- ***Closed- Coil helical Spring***
- helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix ; ***helix angle (α) will be less than 10º***



Helical Springs: (a) Compression Spring (b) Extension Spring

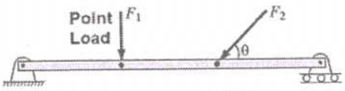


Coil angle – Helix angle

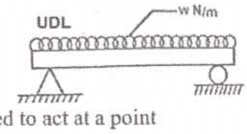
21-11-2022 Lecture notes, by Saloop T S, Lecturer, GPC Kunnamkulam</td><td></td><td></td><td>3</td></tr>
<tr><td>9</td><td>Hoop Stress = p.d /2.t = 50 N/mm²</td><td></td><td></td><td>3</td></tr>
</table>

10

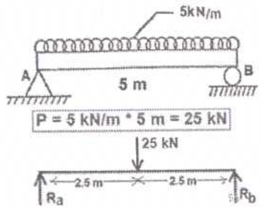
Types of Loading



- The important types of load acting on a beam
  1. ***Concentrated or point load***
  2. ***Uniformly distributed load***
  3. ***Uniformly varying load***

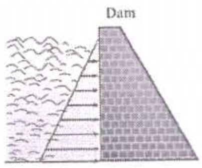


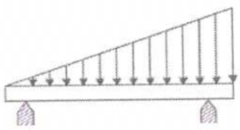
- ***Concentrated or Point Load*** is one which is considered to act at a point
- ***Uniformly Distributed Load (UDL)*** is one in which load is spread over a beam
- The rate of loading ***w*** is uniform along the length
- The rate of loading is expressed as ***w* N/m** run.
- For solving the numerical problems,
  - ***the total UDL is converted into a point load,***
  - ***acting at the centre of UDL***

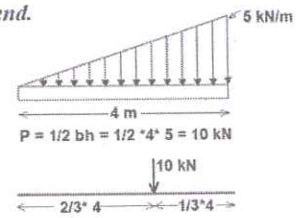


21-11-2022 Lecture notes, by Saloop T S, Lecturer, GPC Kunnamkulam

- ***Uniformly Varying Load (UVL)***
- Load is zero at one end and increases uniformly to the other end
- Rate of loading varies uniformly from point to point
- For solving numerical problems
  - the *total load is equal to* ***the area of the triangle***
  - this *total load is assumed to be acting at the C.G. of the triangle i.e., at a distance of* ***2/3 rd of total length of beam from left end.***







3

PART C

1 | 7

Solution. *Given* : $l_s = 2$ m, $d_s = 3$ mm, $\delta l_s = 0.75$ mm, $E_s = 2.0 \times 10^5$ N/mm²; $l_b = 2.5$ m, $d_b = 2$ mm, $\delta l_b = 4.64$ mm.

Modulus of elasticity of brass, E_b :

From Hooke's law, we know that

$$\delta l = \frac{Pl}{AE}$$

where, δl = Extension, l = Length, A = Cross-sectional area, and E = Modulus of elasticity.

Case I : For steel wire :

$$\delta l_s = \frac{P l_s}{A_s E_s}$$

or,
$$0.75 = \frac{P \times (2 \times 1000)}{\left(\frac{\pi}{4} \times 3^2\right) \times 2.0 \times 10^5}$$

or,
$$P = 0.75 \times \left(\frac{\pi}{4} \times 3^2\right) \times 2.0 \times 10^5 \times \frac{1}{2000} \quad ...(i)$$

Case II : For brass wire:

$$\delta l_b = \frac{P l_b}{A_b E_b}$$

$$4.64 = \frac{P \times (2.5 \times 1000)}{\left(\frac{\pi}{4} \times 2^2\right) \times E_b}$$

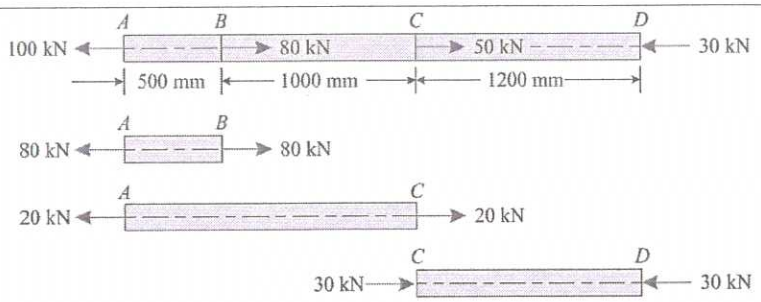
or,
$$P = 4.64 \times \left(\frac{\pi}{4} \times 2^2\right) \times E_b \times \frac{1}{2500} \quad ...(ii)$$

Equating eqns. (*i*) and (*ii*), we get

$$0.75 \times \left(\frac{\pi}{4} \times 3^2\right) \times 2.0 \times 10^5 \times \frac{1}{2000} = 4.64 \times \left(\frac{\pi}{4} \times 2^2\right) \times E_b \times \frac{1}{2500}$$

or,
$$E_b = 0.909 \times 10^5 \text{ N/mm}^2 \quad \textbf{(Ans.)}$$

2 | 7



We know that elongation of the bar,

$$\delta l = \frac{1}{AE}\left[P_1 l_1 + P_2 l_2 + P_3 l_3\right]$$

$$= \frac{1}{500 \times 80}\left[(80 \times 500) + (20 \times 1500) - (30 \times 1200)\right] \text{mm}$$

...(Taking plus sign for tension and minus for compression)

$$= 0.85 \text{ mm} \quad \textbf{Ans.}$$

3

Solution. Refer to Fig. 1.33.

Cross-sectional area of the column

$= 0.4 \times 0.4 = 0.16\ m^2$

Area of steel bars,

$A_s = 4 \times \frac{\pi}{4} \times (0.05)^2$

$= 0.00785\ m^2$

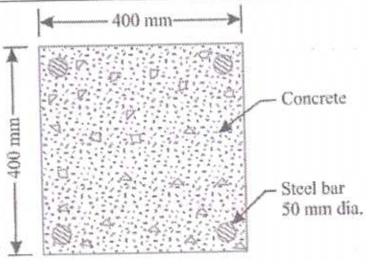


Fig. 1.33

$\therefore$ Area of concrete,

$A_c = 0.16 - 0.00785 = 0.1521\ m^2$

Since the steel bars and concrete shorten by the *same amount* under the compressive load,

$\therefore$ Strain in steel bars = Strain in concrete

or, $e_s = e_c$

$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$ or, $\sigma_s = \sigma_c \cdot \frac{E_s}{E_c} = 15\ \sigma_c$ $(\because\ E_s = 15\ E_c)$

Also, load shared by steel bars + load shared by concrete = 300000 N

or, $P_s + P_c = 300000$ N

or, $\sigma_s \times A_s + \sigma_c \times A_c = 300000$

$15\ \sigma_c \times 0.00785 + \sigma_c \times 0.1521 = 300000$

$\sigma_c\ (15 \times 0.00785 + 0.1521) = 300000$

or, compressive stress in concrete,

$\sigma_c = 1.11 \times 10^6\ N/m^2 = \mathbf{1.11\ MN/m^2}$ **(Ans.)**

7

4

Solution. S.F. calculations:

S.F. between A and C

i.e $S_{A-C} = -1$ kN

S.F. at D *i.e.* $S_D = -1 - 2 \times 2 = -5$ kN

S.F. will change uniformly from -1 kN to -5 kN

S.F. at E *i.e.* $S_E = -5 - 1 = -6$ kN

S.F. between E and B

i.e. $S_{E-B} = -6$ kN

S.F. diagram is shown in Fig. 3.12 (*b*)

B.M. calculations:

B.M. at A *i.e.* $M_A = 0$

B.M. at C *i.e.* $M_C = -1 \times 1 = -1$ kNm

B.M. at D *i.e.* $M_D = -1 \times (1 + 2) - 2 \times 2 \times 2/2 = -3 - 4$

$= -7$ kNm

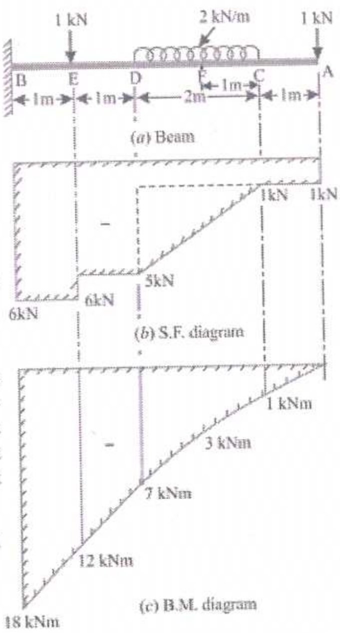


Fig. 3.12

The *B.M.* diagram is a *parabolic curve* for the portion *CD* on which there is U.D.L. Since it is not possible to draw a smooth curve with only two points, so it is necessary to find out the bending moment at least at one more point between *C* and *D*. Let us consider point *F* in the *middle* of *C* and *D*.

B.M. at F *i.e.* $M_F = -1 \times (1 + 1) - 2 \times 1 \times \frac{1}{2} = -2 - 1 = -3$ kNm

B.M. at E *i.e.* $M_E = -1 \times 4 - 2 \times 2 \left(\frac{2}{2} + 1\right) = -4 - 8 = -12$ kNm

B.M. at B *i.e.* $M_B = -1 \times 5 - 2 \times 2\left(\frac{2}{2} + 1 + 1\right) - 1 \times 1 = -5 - 12 - 1 = -18$ kNm

7

5

Solution. To find reactions R_A and R_B, taking moments above A, we get

$$R_B \times 2 = 1 \times 0.5 \times 0.5/2 + 2 \times 0.5 + 5 \times 1.5 + 1 \times 1 \times \left(\frac{1}{2} + 0.5 + 0.5\right)$$

$$= 0.125 + 1 + 7.5 + 1.5 = 10.125$$

or, $R_B = 5.06$ say 5.1 kN

But,

$$R_A + R_B = 1 \times 0.5 + 2 + 5 + 1 \times 1$$
$$= 0.5 + 2 + 5 + 1 = 8.5 \text{ kN}$$

$\therefore \quad R_A = 3.4$ kN

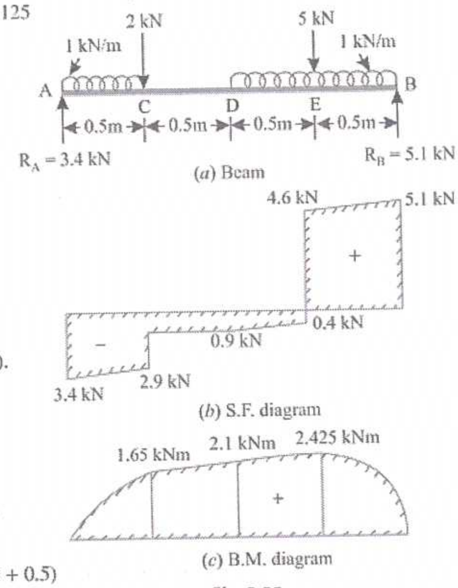


Fig. 3.25

S.F. calculations:

$S_B = 5.1$ kN

$S_E = 5.1 - 1 \times 0.5 - 5 = -0.4$ kN

$S_{D-C} = -0.4 - 1 \times 0.5 = -0.9$ kN

$S_C = -0.9 - 2 = -2.9$ kN

$S_A = -2.9 - 1 \times 0.5 = -3.4$ kN

S.F. diagram is shown in Fig. 3.25 (*b*).

B.M. calculations:

$M_B = 0$

$M_E = 5.1 \times 0.5 - 1 \times 0.5 \times 0.5/2$
$= 2.425$ kNm

$M_D = 5.1 \times 1 - 5 \times 0.5 - 1 \times 1 \times \frac{1}{2}$
$= 5.1 - 2.5 - 0.5 = 2.1$ kNm

$M_C = 5.1 \times 1.5 - 5 \times 1 - 1 \times 1\,(0.5 + 0.5)$
$= 7.65 - 5 - 1 = 1.65$ kNm

7

6

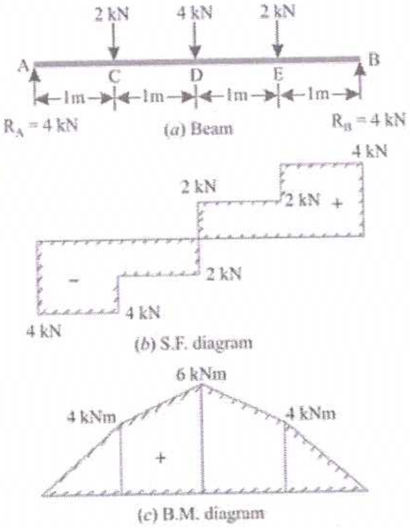


Fig. 3.23

Solution. To determine the support reactions taking moments about A, we get

$$R_B \times 4 = 2 \times 1 + 4\,(1 + 1) + 2\,(1 + 1 + 1)$$
$$= 2 + 8 + 6 = 16$$

$$R_B = \frac{16}{4} = 4 \text{ kN}$$

But, $R_A + R_B = 2 + 4 + 2 = 8$ kN

$\therefore \quad R_A = 8 - R_B = 8 - 4 = 4$ kN

S.F. calculations:

$S_{B-E} = +4$ kN

$S_{E-D} = 4 - 2 = 2$ kN

$S_{D-C} = 2 - 4 = -2$ kN

$S_{C-A} = -2 - 2 = -4$ kN

S.F. diagram is shown in Fig. 3.23 (*b*)

B.M. calculations:

$M_B = 0$

$M_E = 4 \times 1 = 4$ kNm

$M_D = 4\,(1 + 1) - 2 \times 1 = 8 - 2 = 6$ kNm

$M_C = 4\,(1 + 1 + 1) - 2\,(1 + 1) - 4 \times 1$
$= 12 - 4 - 4 = 4$ kNm

$M_A = 4\,(1 + 1 + 1 + 1) - 2\,(1 + 1 + 1) - 4\,(1 + 1) - 2 \times 1$
$= 16 - 6 - 8 - 2 = 0$

7

7	**Solution.** Refer to Fig. 4.7. Width of the beam, $b = 150$ mm $= 0.15$ m Depth of the beam, $d = 250$ mm $= 0.25$ m Maximum bending moment, $M = 750$ kNm Young's modulus of elasticity, $E = 200$ GN/m^2. (*i*) **Maximum stress in the beam:** Moment of inertia, $I = \frac{bd^3}{12} = \frac{0.15 \times 0.25^3}{12} = 0.0001953$ m^4 Distance of neutral axis (N.A.) from the top surface of the beam, $y = \frac{d}{2} = \frac{0.25}{2} = 0.125$ m Using the relation, $\frac{M}{I} = \frac{\sigma}{y}$, we get $\sigma = \frac{M \cdot y}{I} = \frac{750 \times 10^3 \times 0.125}{0.0001953}$ $= 4.8 \times 10^8$ N/m^2 or 480 MN/m^2 *Hence, the maximum stress in the beam* = **480 MN/m^2 (Ans.)** (*ii*) **Radius of curvature, *R*:** Using the relation, $\frac{M}{I} = \frac{E}{R}$, we get $R = \frac{EI}{M} = \frac{200 \times 10^9 \times 0.0001953}{750 \times 10^3}$ = **52.08 m (Ans.)** (*iii*) **Longitudinal stress at a distance of 65 mm from top surface of the beam, σ_1:** Using the relation, $\frac{M}{I} = \frac{\sigma}{y} = \frac{\sigma_1}{y_1}$, we get $\sigma_1 = \frac{M \cdot y_1}{I}$ $= \frac{750 \times 10^3 \times (60 \times 10^{-3})}{0.0001953} \times 10^{-6}$ MN/m^2 ($\because y_1 = 125 - 65 = 60$ mm) = **230.4 MN/m^2 (Ans.)**			7
8	The following assumptions are made while deriving the Euler's formula: 1. The column is initially straight and of uniform lateral dimension. 2. The compressive load is exactly axial and it passes through the centroid of the column section. 3. The material of the column is perfectly homogeneous and isotropic. 4. Pin joints are frictionless and fixed ends are perfectly rigid. 5. The weight of the column itself is neglected. 6. The column fails by buckling alone. 7. Limit of proportionality is not exceeded.			7
9	**Sol.** External diameter, $D = 150$ mm $= 0.15$ m Internal diameter, $d = 100$ mm $= 0.1$ m Length of the column, $l = 10$ m Factor of safety, F.O.S. = 5 $E = 95$ GN/m^2 End conditions: *One end rigidly fixed and the other hinged* $\therefore$ $l_e = \frac{l}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07$ m **Safe compressive load:** Using the relation, $P_{Euler} = \frac{\pi^2 EI}{l_e^2}$, we get $P_{Euler} = \frac{\pi^2 \times 95 \times 10^9 \times \pi/64\,(0.15^4 - 0.1^4)}{(7.07)^2} \times 10^{-3}$ kN $= 374$ kN $\therefore$ Safe load $= \frac{P_{Euler}}{F.O.S.} = \frac{374}{5}$ = **74.8 kN (Ans.)**			7

10	***Determination of load*** Force on the spring, $W = \frac{\pi}{4} \times (120)^2 \times 1 = 11\,310$ N ***Wire diameter*** $\tau = \frac{8WD}{\pi d^3}$ or $70 = \frac{8 \times 11\,310 \times 200}{\pi \times d^3}$ or $d = 43.5$ mm ***Number of coils*** $\delta = \frac{8WD^3 n}{Gd^4}$ or $60 = \frac{8 \times 11\,310 \times 200^3 \times n}{84\,000 \times 43.5^4}$; $n = 24.9 \approx 25$ turns			7
11	Hoop stress = p.d/2.t = 40 N/mm^2 t = 25 mm Longitudinal stress = p.d/4.t = 30 N/mm^2 t = 16.67 mm considering the greater value of the two thickness, t = 25 mm				

12

Let,
T = Maximum twisting torque,
D = Diameter of the shaft,
I_p = Polar moment of inertia,
τ = Shear stress,
C = Modulus of rigidity
θ = The angle of twist (radians), and
l = Length of the shaft.

7

In Fig. 11·1 is shown a shaft fixed at one end and torque being applied at the other end. If a line LM is drawn on the shaft, it will be distorted to LM' on the application of the torque ; thus cross-section will be twisted through angle θ and surface by angle ϕ.

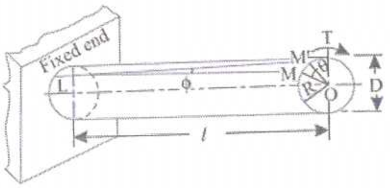


Fig. 11.1

Here, shear strain, $\phi = \frac{MM'}{l}$

Also, $\phi = \frac{\tau}{C}$

$\therefore \quad \frac{MM'}{l} = \frac{\tau}{C}$

or, $\frac{R\theta}{l} = \frac{\tau}{C}$ $\quad \left[\because MM' = R \times \theta,\ R \text{ being radius of the shaft}\right]$

$$\therefore \quad \frac{\tau}{R} = \frac{C\theta}{l} \qquad ...(11\cdot1)$$

Refer to Fig. 11·2. Consider an elementary ring of thickness dx at a radius x and let the shear stress at this radius be τ_x.

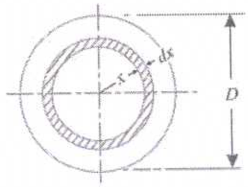


Fig. 11.2

The turning force on the elementary ring

$= \tau_x \,.\, 2\pi x \,.\, dx$

Turning moment due to this turning force,

$dT = \tau_x \,.\, 2\pi x \,.\, dx \times x$

To get total turning moment integrating both sides, we get

$$\int dT = \int_0^R \tau_x \,.\, 2\pi x\, dx \times x$$

or, $$\int dT = 2\pi \int_0^R \tau_x \,.\, x^2 \,.\, dx = 2\pi \int_0^R \frac{\tau \,.\, x}{R} \,.\, x^2 \,.\, dx \qquad \left[\because \frac{\tau}{R} = \frac{\tau_x}{x} \text{ or } \tau_x = \frac{\tau . x}{R}\right]$$

$$= 2\pi \frac{\tau}{R} \int_0^R x^3\, dx$$

or, $$T = 2\pi \frac{\tau}{R} \left|\frac{x^4}{4}\right|_0^R = 2\pi \frac{\tau}{R} \cdot \frac{R^4}{4}$$

$$T = \tau \,.\, \frac{\pi R^3}{2} = \tau \cdot \frac{\pi}{16} D^3 \qquad ...(\textit{Strength of solid shaft})$$

or, $$T = \frac{\tau}{R} \cdot \frac{\pi R^4}{2} = \frac{\tau}{R} I_p \qquad \left[\because I_p = \frac{\pi}{32} D^4 = \frac{\pi}{2} R^4\right]$$

$$\therefore \quad \frac{T}{I_p} = \frac{\tau}{R} \qquad ...(11\cdot2)$$

From eqns. (11·1) and (11·2), we have

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l} \qquad ...(11\cdot3)$$

This is called **torsion equation.**