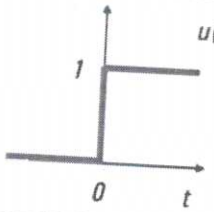
(16) (9) Nov 23

SCORING INDICATORS

COURSE NAME : SIGNALS AND SYSTEMS (SET2)
COURSE CODE : TED (21) 5201

QID: 2109230106

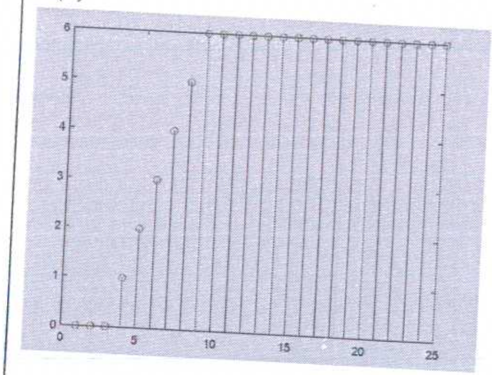
Q No	Scoring Indicators	Split score	Sub Total	Total score
	PART A			9
I. 1	The signal whose energy is finite and average power is zero	1	1	
I. 2	 The time axis should be marked. The magnitude should be 1. Step at t = 0 should be shown.	1	1	
I. 3	The signals that follow the even symmetry with respect to the y-axis(magnitude) are called even signals. The signals which follow the relation *x(t)* = *x(-t)* for all *t* or *x(n)* = *x(-n)* for all *n* are called even signals **One of these two definitions**	1	1	
. 4	$x(n) = \begin{cases} 0, for\ n < 0 \\ 1, for\ 0 \leq n \leq 5 \\ 0, for\ n > 5 \end{cases}$ **Or** $x(n) = sin(\omega n) for\ all\ n$ — In any one of the forms *x(n)* should be represented as a function of *n*	1	1	
I. 5	The system whose output at any time depends only on present and past values of input but not on future inputs	1	1	
I. 6	Any one equation in the form $y(n) = a.x(n) + b.x(n-1) + c.y(n-1) + \cdots$ $a, b, c, and\ etc. can\ be\ numbers\ also.$	1	1	
I. 7	DTFT			
I. 8	2.f_{max}	1	1	
I. 9	$e^{-at}u(t)$,	1	1	
	e^{-at} can also be give full mark	1	1	
	PART B			24
II. 1	Causal signals: the signals which does not exists or has zero magnitude for negative values of time (t or n) are called causal signals. A continuous-time signal is said to be causal if x(t) = 0 for t < 0. Anti-causal signals: the signals that do not exist or have zero	1 x 3	3	

<table>
<tr><td></td><td colspan="2">magnitude for positive values of time (t or n) are called anti-causal signals. A continuous-time signal is said to be causal if $x(t) = 0$ for $t > 0$.

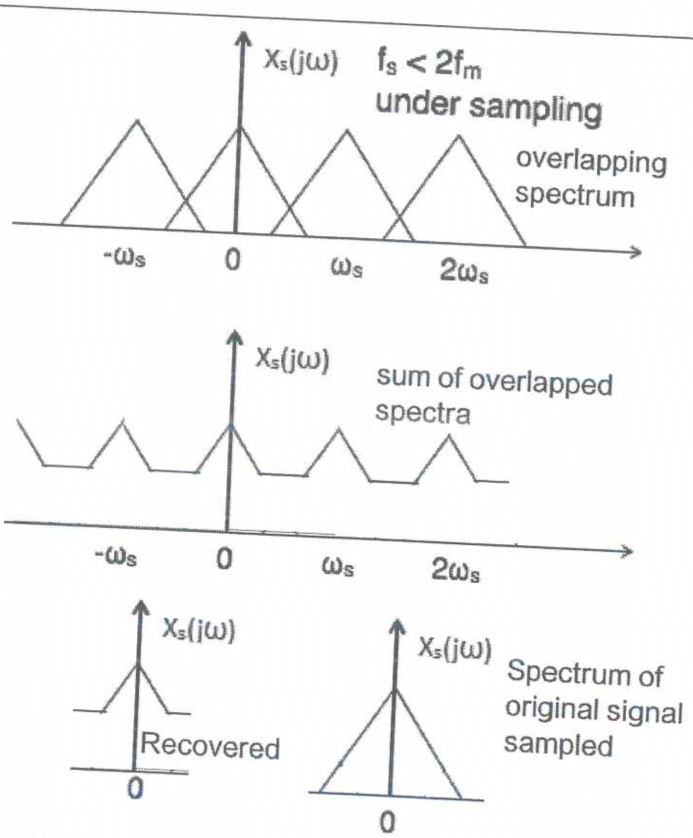
Non-causal signals: all the signals which are not causal are called non-causal signals. Non-causal signals include anti-causal signals and the signals that exist for both positive and negative values of time.</td><td></td><td></td><td></td></tr>
<tr><td>II. 2</td><td colspan="2">Time scaling is either compression or expansion. (1 marks)
Explanation 2 marks
Replacing ‘t’ with ‘At’ in a signal will cause expansion of the signal if ‘A’ is a positive constant with a magnitude less than 1.
If ‘A’ is a positive constant with a magnitude greater than 1, it will cause time compression of the signal.
If A is negative in addition to expansion or compression the signal will be time reversed.

(full marks may be given even if time reversal is not mentioned. Diagrams are not necessary)</td><td>1+2</td><td>3</td><td></td></tr>
<tr><td>II. 3</td><td colspan="2">Plot of any 3 continuous time signals from the following, with time axis and amplitude clearly labeled.
i. Unit step signal [u(t)]
ii. Unit impulse signal [$\delta(t)$]
iii. Unit ramp signal [r(t)]
iv. Unit parabolic signal [p(t]
v. Sinusoidal signal
vi. Signum function
vii. Real exponential signal
viii. Rectangular pulse
ix. Triangular pulse</td><td>1x 1</td><td>3</td><td></td></tr>
<tr><td>II. 4</td><td>Periodic Signal</td><td>Aperiodic Signal</td><td></td><td></td><td></td></tr>
<tr><td></td><td>A definite pattern of repetition at regular interval</td><td>No pattern of repetition</td><td>1</td><td>3</td><td></td></tr>
<tr><td></td><td>A finite value of fundamental period.
$x(t+T) = x(t)$ or
$x(n+N) = x(n)$</td><td>Ideally period is infinite</td><td>1</td><td></td><td></td></tr>
<tr><td></td><td>eg:- sin(ωt)
sinusoids</td><td>eg:- u(t)
rectangular pulse, triangular pulse, and etc.</td><td>1</td><td></td><td></td></tr>
<tr><td>II. 5</td><td>CT System</td><td>DT System</td><td></td><td></td><td></td></tr>
<tr><td></td><td>Input and output are CT signals</td><td>Input and output are DT signals</td><td>1 x3</td><td>3</td><td></td></tr>
</table>

	Described by a differential equation	Described by a difference equation		
	Impulse response is denoted as h(t)	Impulse response is denoted as h(n)		
	No such parameter, any continuous time signal can be processed	Sampling frequency of input is an important parameter to process		
	eg:- Audio or video Amplifiers	eg:- microprocessors, microcontrollers, DSP processors		
II. 6	The response of a system to impulse is called impulse response. It is the output of the system when the input is the unit impulse signal The impulse response of a causal system is a causal signal. That means the impulse response has non-zero amplitude only for the positive values of time [or h(t) = 0 for t<0)] For a stable system the impulse response is a bounded signal. That means the impulse response is absolutely integrable.		1+1+1	3
II. 7	For a system to be invertible, there should be a system such that which recovers the input from the output of the original system. a) not invertible, since the original input cannot be recovered after squaring the signal. The sign of the signal cannot be recovered b) invertible. The original system only introduces a time delay. The inverse system can be implemented as a system introducing a equal time advancement; ie., y(n) = x(n+2)		1½ x 2	3
II. 8	**EXISTENCE OF FOURIER TRANSFORM** There are a set of sufficient conditions called Dirichlet conditions under which a continuous-time signal x(t) is guaranteed to have a Fourier transform: 1. x(t) is absolutely integrable: $\int_{-\infty}^{\infty}\lvert x(t)\rvert dt < \infty$ 2. x(t) has a finite number of maxima and minima in any finite interval. 3. x(t) has a finite number of discontinuities in any finite interval, and each of these discontinuities is finite.		1x3	3
II.9	Statement 1 marks and explanation 2 mark The final value theorem states that if X(s) is the Laplace transform of x(t) then $$x(\alpha) = \lim_{s\to 0} sX(s)$$ Final value theorem is applied to find $x(\alpha)$ when x(t) cannot be evaluated at $t = \alpha$ Final value theorem is applied when all the poles of sX(s) lies		1+2	3

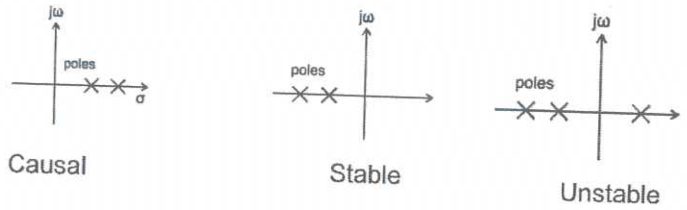
	on the left half of s-plane			
II.10	Any 3 points **PROPERTIES OF ROC OF LAPLACE TRANSFORM** ❖ ROC contains strip lines parallel to jω axis in s-plane. ❖ If x(t) is absolutely integral and it is of finite duration, then ROC is entire s-plane ❖ If x(t) is a right sided sequence then ROC: Re{s} > σ_o. ❖ If x(t) is a left sided sequence then ROC: Re{s} < σ_o. ❖ If x(t) is a two-sided sequence then ROC is the combination of two regions.	1x3		
	PART C			
III.	a) Continuous-time signals and discrete-time signals b) Even signals and odd signals c) Periodic signals and aperiodic signals d) Energy signals and power signals e) Real signals and complex(imaginary) signals f) Deterministic signals and non-deterministic signals List 2 marks Explanation 5 marks	2+5	7	42
IV	x(n)  The signal has zero magnitude for t < 0, therefore the signal is causal Drawing the signal 5 marks Checking causality 2 marks	5+2	7	
V	For checking linearity from a difference equation, linear combination of output is compared with the output when the input is a linear combination. a) let $y_1(t) = T[x_1(t)] = x_1^2(t)$ and $y_2(t) = T[x_2(t)] = x_2^2(t)$ Then the linear combination of outputs, is $A.y_1(t) + B.y_2(t) = A.x_1^2(t) + B.x_2^2(t)$………(1) Now the output when input is a linear combination of $x_1\ (t)$ and $x_2\ (t)$, is	3+4 3 marks	7	

	$T[A.x_1(t)+B.x_2(t)] = (A.x_1(t)+B.x_2(t))^2$ $A^2.x_1^2(t)+B^2.x_2^2(t)+A.B.x_1(t).x_2$ ....(2) Since eqn.(1) ≠ eqn.(2) the system is not linear b) let $y_1(t)=T[x_1(t)]$ and $y_2(t)=T[x_2(t)]$ when y(t) in the difference equation is replaced with a linear combination of outputs, the LHS becomes $\frac{d}{dt}(A.y_1(t)+B.y_2(t))+A.y_1(t)+B.y_2(t)$ $=\frac{d}{dt}(A.y_1(t))+A.y_1(t)+\frac{d}{dt}(B.y_2(t))+B.y_2(t)$ $=A.\left(\frac{dy_1(t)}{dt}+y_1(t)\right)+B.\left(\frac{dy_2(t)}{dt}+y_2(t)\right)$ $=A.\left(x_1(t)\frac{dx_1(t)}{dt}\right)+B.\left(x_2(t)\frac{dx_2(t)}{dt}\right)$ ....(3) when x(t) in the difference equation is replaced with a linear combination of inputs, the RHS becomes $(A.x_1(t)+B.x_2(t)).\frac{d}{dt}(A.x_1(t)+B.x_2(t))$ $=A^2.\left(x_1(t)\frac{dx_1(t)}{dt}\right)+B^2.\left(x_2(t)\frac{dx_2(t)}{dt}\right)+$ $A\ .B.\left(x_1(t)\frac{dx_2(t)}{dt}\right)+A\ .B.\left(x_2(t)\frac{dx_1(t)}{dt}\right)$ ....(4) Since eqn. (3) ≠ eqn. (4) the system is not linear	4 marks		
VI	Any 4 properties of 1. Linearity 2. Time invariance 3. Causality 4. Possessing Memory (Static or Dynamic) 5. Stability 6. Invertibility Listing 1 marks Description 1½ marks each	1+1½ x 4	7	
VII	Statement 2 marks 5 marks for diagram and explanation Sampling theorem: A bandlimited continuous time signal with maximum frequency f_m can be represented using its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component, f_m. i.e., $f_s \geq 2.f_m$	2+5	7	

	 			
VIII	Statement 2 marks Explanation 5 marks The convolution property states that the convolution of signals in time domain is equivalent to multiplication of Fourier transform of individual signals. That means If $x(t) \xrightarrow{F} X(j\omega)$ and $h(t) \xrightarrow{F} H(j\omega)$ then $x(t) * h(t) \xrightarrow{F} X(j\omega).H(j\omega)$ The convolution property is used for evaluating the spectrum output signal when the input and impulse response are known.	2+5	7	
IX	**Time Scaling** If $x(t)$ is periodic with period T, then $x(at)$ will be periodic with period T/a; a>0 If $x(t) \xleftrightarrow{FS} c_n$ Then $x(at) \xleftrightarrow{FS} c_n$ Thus, after time scaling FS coefficients are the same. But, the spacing between the frequency components changes from ω_0 to $a\omega_0$ or from $1/T$ to a/T **Convolution** If $x(t) \xleftrightarrow{FS} c_n$ and $y(t) \xleftrightarrow{FS} d_n$ Then $x(t) * y(t) \xleftrightarrow{FS} Tc_n d_n$ Hence, the convolution in time domain leads to multiplication of Fourier serie coefficients in Fourier series domain.	1 2	 7	

	Frequency Shifting If $x(t) \xleftrightarrow{FS} c_n$ Then according to frequency shifting property, $e^{jm\omega_0 t} x(t) \xleftrightarrow{FS} C_{(n-m)}$ **Differentiation in Time** According to this property, if $x(t) \xleftrightarrow{FS} c_n$ Then $\frac{d}{dt} x(t) \xleftrightarrow{FS} (jn\omega_0) c_n$	2 2		
X	a) $e^{-at}u(t)$ $X(j\omega) = \int_{-\alpha}^{\alpha} e^{-at}\, u(t) e^{-j\omega t} dt = \int_{0}^{\alpha} e^{-at}\, e^{-j\omega t} dt$ $= \int_{0}^{\alpha} e^{-(a+j\omega)t}\, dt = \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\alpha} = \frac{0-1}{-(a+j\omega)} = \frac{1}{a+j\omega}$ b) $\delta(t)$ $X(j\omega) = \int_{-\alpha}^{\alpha} \delta(t)\, e^{-j\omega t} dt = e^{-j\omega 0} = 1$ c) $\cos(\omega_0 t)$ $X(j\omega) = \int_{-\alpha}^{\alpha} \cos(\omega_0 t)\; e^{-j\omega t} dt$ $= \int_{0}^{\alpha} \frac{e^{-j\omega_0 t} + e^{-j\omega_0 t}}{2j} e^{-j\omega t} dt$ $= \frac{1}{2j} \cdot \left(F\left[e^{-j\omega_0 t}\right] + F\left[e^{j\omega_0 t}\right]\right)$ $= \frac{1}{2j}\left(2\pi.\,\delta(\omega+\omega_0) + 2\pi.\,\delta(\omega-\omega_0)\right)$ $-j\pi\left(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right)$	2 2 3	7	
XI	a) $\frac{1}{(s-3)(s-2)}$ let $\frac{A}{s-3} + \frac{B}{s-2} = \frac{A}{s-3} + \frac{B}{s-2}$ then $\frac{A(s-2)+B(s-3)}{s-3} = \frac{A}{s-3} + \frac{B}{s-2}$ or $A(s-2) + B(s-3) = 1$ when $s = 2, A(2-2) + B(2-3) = 1 \rightarrow -B = 1$ or $\rightarrow B = -1$ when $s = 3, A(3-2) + B(3-3) = 1 \rightarrow A = 1$ Now the expression becomes $\frac{1}{s-3} - \frac{1}{s-2}$ The inverse Laplace transform is $(e^{3t} - e^{2t})u(t)$ b) $\frac{s-a}{(s-a)^2+b^2}$ the Laplace transform of $\cos(bt)$ is $\frac{s}{s^2+b^2}$	3 2	7	

	Using first shifting property of Laplace transform Laplace transform of $e^{at}\sin(bt)$ is $$\frac{s-a}{(s-a)^2+b^2}$$ Inverse Laplace transform is $e^{at}\sin(bt)$ c) $\frac{1}{s^2+2bs+b^2}=\frac{1}{(s+b)^2}$ the Laplace transform of $\frac{1}{s^2}$ is $$tu(t)$$ using first shift property of Laplace transform the Laplace transform of $te^{-at}u(t)$ is $\frac{1}{(s+b)^2}$. Therefore, the answer is $te^{-at}u(t)$		2		
XII	Function	Laplace Transform		7	
	a) $\cos(bt)$	$\frac{s}{s^2+b^2}$	1		
	b) e^{-at} u(t)	$\frac{1}{s+a}$	1		
	c) $\sin(bt)$	$\frac{b}{s^2+b^2}$	1		
	d) $\delta(t)u(t)$	1	1		
	e) r(t)	$\frac{1}{s^2}$	1		
	f) p(t)	$\frac{2}{s^3}$	1		
	g) u(t)	$\frac{1}{s}$	1		
XIII	Time Scaling: $$f\left(\frac{t}{a}\right)\overset{\mathcal{L}}{\longleftrightarrow} aF(as)$$ Linearity $$f_1(t)\overset{L.T.}{\longrightarrow}F_1(s)\ \ with\ ROC=R_1$$ $$f_2(t)\overset{L.T.}{\longrightarrow}F_2(s)\ \ with\ ROC=R_2$$ $$af_1(t)+bf_2(t)\overset{L.T.}{\longrightarrow}aF_1(s)+bF_2(s):\ ROC=R_1\cap R_2$$ Time Shifting $$f(t)\overset{L.T.}{\longrightarrow}F(s)\ \ with\ ROC=R$$ $$f(t-t_0)\overset{L.T.}{\longrightarrow}e^{-st_0}F(s):ROC=R$$		1 + 2 + 2 +	7	

	Initial Value Theorem Initial value theorem is applied when in Laplace transform the degree of the numerator is less than the degree of the denominator $f(0) = \lim_{s \to \infty} sF(s)$	2		
XIV	Definition 2 marks illustration and explanation 5 marks Region of Convergence (ROC) is defined as the set of points in s-plane for which the Laplace transform of a function x(t) converges. In other words, the range of Re(s) (i.e. σ) for which the function X(s)converges is called the region of convergence.  	2+5	7	