SET 2

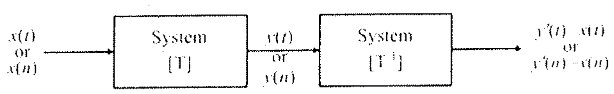
SET 2

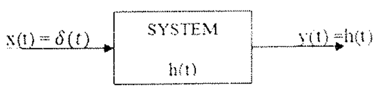
SCORING INDICATORS

COURSE NAME: SIGNALS AND SYSTEMS
COURSE CODE: 5201

QID: 2109230108

Q No	Scoring Indicators	Splitscore	Sub Total	Total score
	PART A			9
I. 1	Speech and audio signal processing, multimedia processing (image and video), biological signal analysis (ECG, EEG, X-ray etc), underwater acoustics, communication applications, industrial control and automation, RADAR, astronomy, seismography	0.5*2	1	
I. 2	Continuous time signals are defined for all values of time whereas discrete time signals are defined only at discrete instants of time. Discrete time signal is obtained by sampling continuous-time signals at regular intervals.		1	
I. 3	A system is called a BIBO (bounded input bounded output) stable system, if and only if every bounded input produces a bounded output.		1	
I. 4	A discrete time system is one which operates on a discrete-time input signal and produces a discrete-time output signal		1	
I. 5	The Fourier representation provides valuable information about the frequency content of a signal. By decomposing the signal into its frequency components, we can analyse its spectral content, identify dominant frequencies, filter out unwanted components, and manipulate signals in the frequency domain.		1	
I. 6	Fourier Transform (continuous and discrete Fourier transform)		1	
I. 7	Aliasing can be avoided by sampling the continuous -time signal at a frequency greater than twice the maximum frequency present in the signal		1	
I. 8	In the Laplace transform representation of a signal: **poles are the roots of the denominator polynomialor** (in otherwords, **poles are the values of s for which the trasform becomes infinite**)		1	
I. 9	**Complex frequency domain** or **S-domain**		1	

<table>
<tr><td></td><td>PART B</td><td></td><td></td><td>24</td></tr>
<tr><td>II. 1</td><td>Fourier Series
1. Gives the frequency content of a periodic signal
2. Discrete Frequency spectrum
Fourier Transform
1. Gives the frequency information of an aperiodic signal
2. Continuous frequency spectrum</td><td>1.5*2</td><td>3</td><td></td></tr>
<tr><td>II. 2</td><td>Invertibility
If a system has a unique relationship between its input and output, the system is called the invertible system. In other words, a system is said to be an invertible system only if an inverse system exists which when cascaded with the original system produces an output equal to the input of the first system.


y(t)=5x(t) is an invertible system
$y(t) = 5x(t)$
Let, $x(t) = 3$, then the output of the system is,
$y(t) = 5 \times 3 = 15$
Let, $x(t) = -3$, then the output of the system is,
$y(t) = 5 \times (-3) = -15$
Hence, different inputs leads to different outputs. Therefore, the system is invertible system.
Here the original system has a gain of 5. A system with a gain 1/5 acts as the inverse system for the given system.</td><td>2(expln)
+
1(eg.)</td><td>3</td><td></td></tr>
<tr><td>II. 3</td><td>When the unit impulse function is applied as input to an LTI system, then the output is nothing but impulse response h(t).


By knowing the impulse response of a system, we can predict its response to any input signal by convolving the input signal with the impulse response. The impulse response is used to study various properties of the system such as causality, stability, dynamicity etc</td><td>1.5(definition)
+
1.5(significance)</td><td>3</td><td></td></tr>
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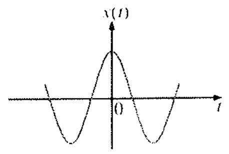
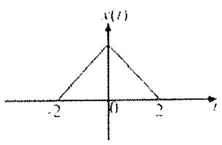
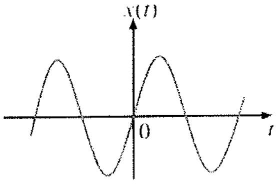
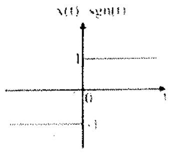
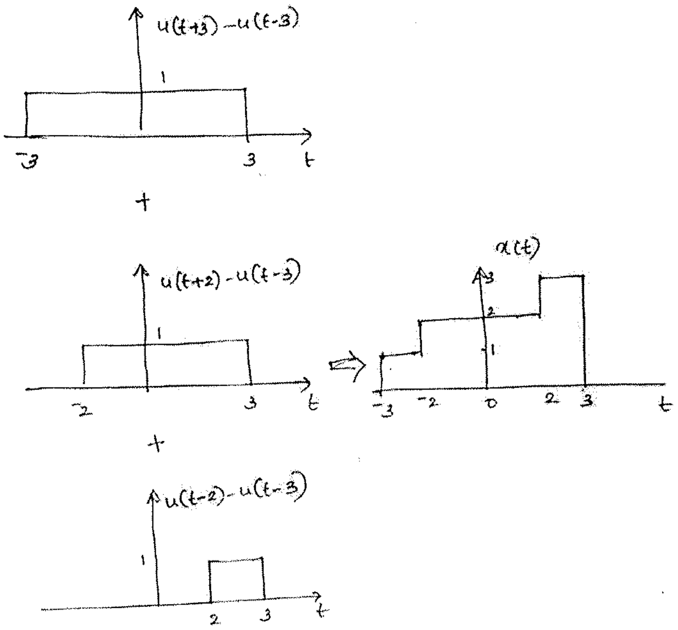
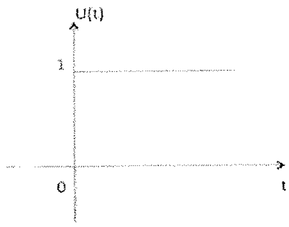
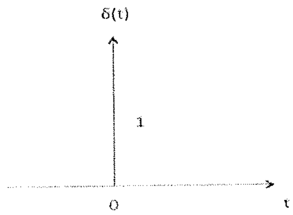
II. 4	Here $y(n) = x(n^2)$. If n = 2, then, $y(2) = x(4)$ The output depends upon future inputs. Hence system is noncausal.		3	
II. 5	Convolution operation to evaluate the output of a system involves the following steps 1. **Folding or time reversal** 2. **Shifting** 3. **Multiplication** 4. **Summation or integration**.	**0.75*4**	3	
II. 6	Every signal $x(t)$ of period T satisfying following conditions known as Dirichlet's conditions, can be expressed in the form of Fourier series 1. The signal should have only a finite number of maxima and minima over a given period. 2. The signal must possess only a finite number of discontinuities over a given period. 3. The signal must be absolutely integrable over a given period i.e., $\int_0^T \lvert x(t) \rvert dt < \infty$		3	
II. 7	**Even Signal** • A signal which is symmetrical about the vertical axis or time origin is known as **even signal** or **even function**. Therefore, the even signals are also called the **symmetrical signals**. • A DT signal x(n) is said to be an even signal if x(-n)=x(n) and a CT signal x(t) is said to be an even signal if x(t)=x(-t) Ex, cosine wave, rectangular signal, triangular signal   **Odd signal** • A signal that is anti-symmetrical about the vertical axis is known as odd signal or **odd function**. Therefore, the odd signals are also called the **antisymmetric signals**. • A DT signal x(n) is said to be an odd signal if x(-n) =-x(n)	**2(expln) +1(eg.)**	3	

	and a CT signal x(t) is said to be an odd signal if x(-t) =-x(t) Ex: signum function, sine wave  			
II. 8	Given a continuous-time signal x(t), the **Fourier Transform** of the signal is given by $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ Analysis The **Inverse Fourier Transform** of the signal is given by $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$ Synthesis	**1.5*2**	**3**	
II.9	Linearity, Time Shifting, Frequency Shifting, Time Scaling, Time Inversion, Convolution, Multiplication in Time Domain, Symmetry	**1*3** ***(any three)***	**3**	
II.10	**Properties of ROC** 1. The ROC of X(s) consists of strips parallel to the jω axis in the s-plane. 2. The ROC does not contain any poles. 3. If x(t) is of finite duration and is absolutely integrable, then the ROC is the entire s-plane. 4. If x(t) is a right sided signal, that is x(t) = 0 for $t<t_0<\infty$ then the ROC is of the form Re(s)> a_{max}, where a_{max} equals the maximum real part of any of the poles of X(s). 5. If x(t) is a left sided, that is x(t) = 0 for $t>t_1>-\infty$, then the ROC is of the form Re(s)< a_{min} , where a_{min} equals the minimum real part of any of the poles of X(s). 6. If x(t) is a two-sided signal, than the ROC is of the form a_1<Re(s)< a_2.	**1*3** ***(any three)***	**3**	

	PART C			42
III.	 $x(t) = u(t+3) - u(t-3) + u(t+2) - u(t-3) + u(t-2) - u(t-3)$ ie, $x(t) = u(t+3) + u(t+2) + u(t-2) - 3u(t-3)$	**5(Finding elementary signals)** + **2(Expression)**	**7**	
IV.	**Elementary signals:** unit step, unit impulse, ramp, parabolic, exponential, rectangular, triangular, sinusoidal, signum function Unit Step Function Unit step function is denoted by u(t). It is defined as u(t) = $\begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases}$ 	***Any four*** **4(Expression)** + **3(fig)**	**7**	

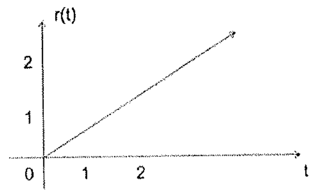
Unit Impulse Function

Impulse function is denoted by δ(t). and it is defined as $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$



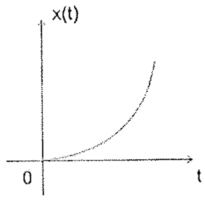
Ramp Signal

Ramp signal is denoted by r(t), and it is defined as $r(t) = \begin{cases} t & t \geqslant 0 \\ 0 & t < 0 \end{cases}$



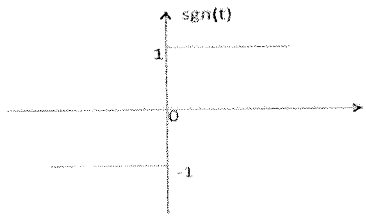
Parabolic Signal

Parabolic signal can be defined as $x(t) = \begin{cases} t^2/2 & t \geqslant 0 \\ 0 & t < 0 \end{cases}$



Signum Function

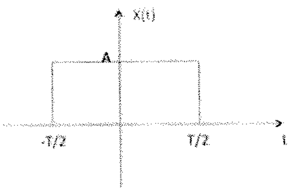
Signum function is denoted as sgn(t). It is defined as $sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$



Rectangular Signal

Let it be denoted as x(t) and it is defined as

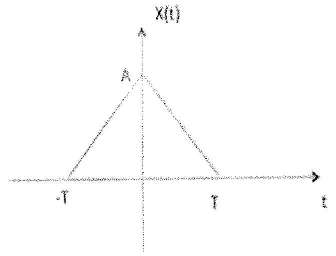
$$x(t) = A\, rect\left[\frac{t}{T}\right]$$



Triangular Signal

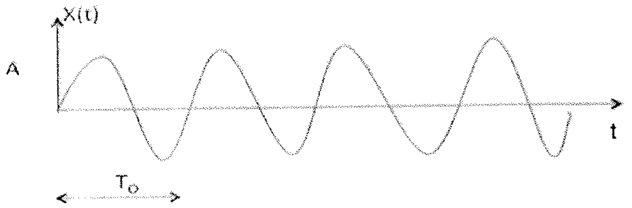
It is denoted as

$$x(t) = A\left[1 - \frac{|t|}{T}\right]$$



Sinusoidal Signal

Sinusoidal signal is in the form of $x(t) = A\cos(w_0 t \pm \phi)$ or $A\sin(w_0 t \pm \phi)$



Where, A is the amplitude,

ω_0 is the frequency in radians per sec and

ϕ is the phase shift in radians

$T_0 = 2\pi/\omega_0$ is the period

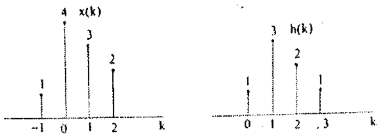
V.

$$x(n) = \{1, 4, 3, 2\}; h(n) = \{1, 3, 2, 1\}$$
$$\uparrow$$

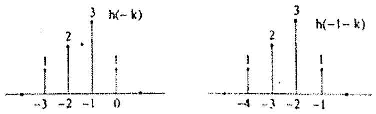
Solution:

Step 1: The sequence $x(n)$ starts at $n_1 = -1$ and $h(n)$ starts at $n_2 = 0$. Therefore the starting time for evaluating the output sequence $y(n)$ is $n = n_1 + n_2 = -1 + 0 = -1$.

Step 2: Express both sequences in terms the index k.



Step 3: Fold $h(k)$ about $k = 0$, to obtain $h(-k)$



As starting time to evaluate $y(n)$ is -1, shift $h(k)$ by one unit to left to obtain $h(-1-k)$.

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k)$$

Multiply the two sequences $x(k)$ and $h(-1-k)$ element by element and sum the products

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k)$$
$$= (0)(1) + (0)(2) + 0(3) + (1)(1) + 4(0) + 3(0) + 2(0) = 1$$

Increment the index by 1, i.e., n to zero, shift the sequence to right to obtain $h(-k)$ and then multiply the two sequences $x(k)$ and $h(-k)$ element by element and sum the products we get

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k)$$
$$= (0)(1) + (0)(2) + 1(3) + 4(1) + 3(0) + 2(0) = 7$$

Similarly,

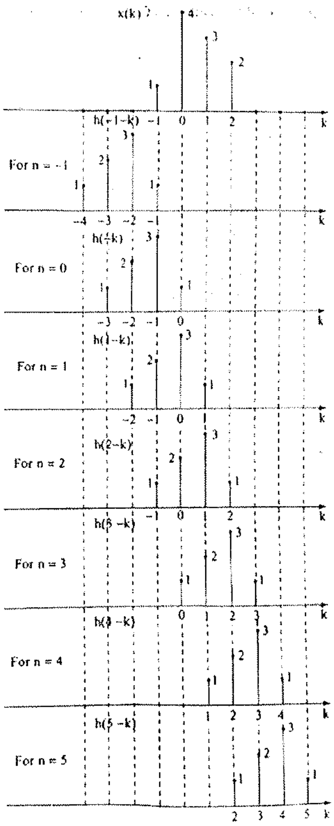
$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k)$$
$$= 0(1) + 1(2) + 4(3) + 3(1) + 2(0) = 17$$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k)$$
$$= 1(1) + 4(2) + 3(3) + 2(1) = 20$$

$$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k)$$
$$= 1(0) + 4(1) + 3(2) + 2(3) + 0(1) = 16$$

$$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k)$$
$$= 3.1 + 2.2 = 7$$

$$y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k) = 2.1 = 2$$



$y(n) = \{1, 7, 17, 20, 16, 7, 2\}$
$\uparrow$

4(Procedure) + 3(Calculations and final output)

7

VI.	(i). **Linear and Non-Linear systems** • A system is said to be linear if it obeys the principle of homogeneity and principle of superposition. • Therefore, for a continuous-time linear system, $[ay_1(t) + by_2(t)] = T[ax_1(t) + bx_2(t)] = aT[x_1(t)] + bT[x_2(t)]$ • Also, for a discrete-time linear system, $[ay_1(n) + by_2(n)] = T[ax_1(n) + bx_2(n)] = aT[x_1(n)] + bT[x_2(n)]$ Hence, we can say that a system is linear if the output of the system due to weighted sum of inputs is equal to the weighted sum of outputs. A system is said to be a non-linear system if it does not obey the principle of homogeneity and principle of superposition. $y(n)=nx(n)$ and $y(t)=t^2x(t)$ are some examples of linear system. (ii). **Time invariant and Time varying systems.** • If the input and output characteristics of a system do not change with time, the system is called the **time-invariant system**. Time invariance means that the behaviour of the system does not depend on the time at which the input is applied to the system. • A system whose input and output characteristics change with the time is known as **time-variant system**. Ex: y(t)=x(n-1) is a TIV system whereas y(n)=nx(n-1) is not TIV (iii). **Causal and non-causal systems.** • A system whose output or response at any time instant (t) depends only on the present and past values of the input but not on the future values of the input is called the **causal system**. Ex: y(t)=x(t-3), y(n)=nx(n) • A system whose output or response at any time instant (t) depends also on the future values of the input is called the **non-causal system**. Ex: y(t)=x(t+3), y(n)=x(n^2) (iv). **Stable and unstable systems.** • A system is called a BIBO **(bounded input bounded output) stable system** or simply **stable system**, if and only if every bounded input produces a bounded output. • If the impulse response of the system is absolutely integrable for	*any three* **4(Explntn)** **+** **3(Eg.)**	7	

a continuous time system or absolutely summable for a discrete time system, then the system is a stable system.

- If a system does not satisfy the BIBO stability condition, the system is called the unstable system. We can say that a system is unstable even if one bounded input generates an unbounded output.

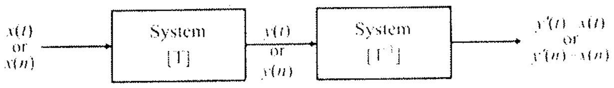
Example 1:
$y(t) = x^2(t)$
Let the input is u(t) (unit step bounded input) then the output $y(t) = u^2(t) = u(t)$ = bounded output. Hence, the system is stable.
Example 2: $y(t) = \int x(t)dt$
the input is u (t) (unit step bounded input) then the output $y(t) = \int u(t)dt$= ramp signal (unbounded because amplitude of ramp is not finite it goes to infinite when $t \rightarrow$ infinite). Hence, the system is unstable.

(v). **Static and dynamic systems.**

- The static system is also called **the memoryless system**. For a static or memoryless system, the output of the system at any instant of time depends only on the input applied at that instant of time, but not on the past or future values of the input.
  Ex: $y(t)=x(t)$
- A system whose response or output depends upon the past or future inputs in addition to the present input is called the **dynamic system**. The dynamic systems are also known as **memory systems**.
  Ex: $y(t)=x(t+1)$

(vi). **Invertible and non-invertible systems.**

If a system has a unique relationship between its input and output, the system is called the invertible system. In other words, a system is said to be an invertible system only if an inverse system exists which when cascaded with the original system produces an output equal to the input of the first system.



$y(t)=5x(t)$ is an invertible system

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$$y(t) = 5x(t)$$

Let, $x(t) = 3$, then the output of the system is,

$$y(t) = 5 \times 3 = 15$$

Let, $x(t) = -3$, then the output of the system is,

$$y(t) = 5 \times (-3) = -15$$

Hence, for different inputs, there is different outputs. Therefore, the system is invertible system.

but, $y(t)=5x^2(t)$ is not an invertible system

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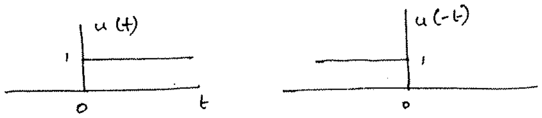
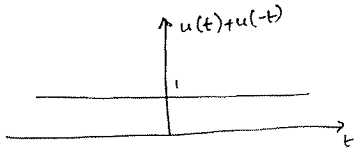
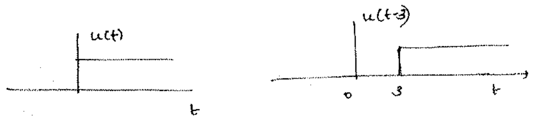
<td>VII.</td>
<td>

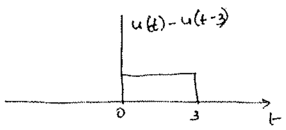
(i) $u(t)+u(-t)$





(ii) $u(t)-u(t-3)$



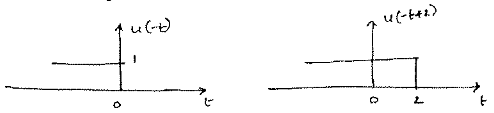


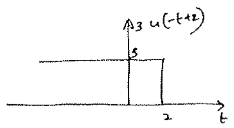
(iii) $3u(-t+2)$

$u(-t+2) \Rightarrow u(-(t-2))$

First reverse $u(t)$ and then right shift by 2 units

Also multiply $u(-(t-2))$ by 3 to get $3u(-t+2)$





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<td>1+2+2+2</td>
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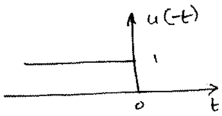
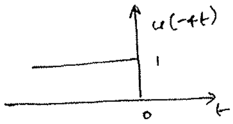
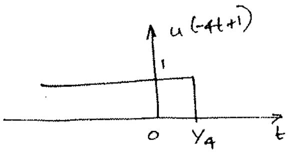
<tr><td></td><td>(iv) $u(-4t+1)$

$u(-4t+1) \Longrightarrow u(-4(t-1/4))$

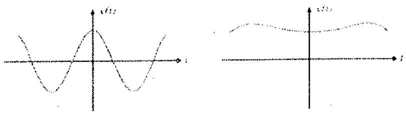
Reverse u(t) first, Then Scale by 4 and then do right shifting by 1/4 units



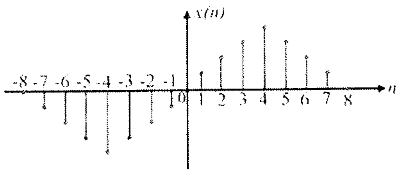



</td><td></td><td></td><td></td></tr>
<tr><td>VIII.</td><td>(1) Continuous time and discrete time signal

• The signals which are defined for every instant of time are called as continuous time signals. The continuous-time signals are also called as analog signals. In case of continuous time signals, the independent variable is time.



• Those signals which are defined only at discrete instants of time are called as discrete time signals. The amplitude of discrete time signals is continuous but these signals are discrete in time. The amplitude of a discrete time signal between two time instants is not defined.



(2) Deterministic and random signals

• Deterministic Signals – A deterministic signal is the one that exhibits no uncertainty of amplitude and phase at any instant of time. These signals have a regular pattern.
Sine wave, exponential signals, square wave, etc. are the examples of deterministic signals.

• Random Signals – A signal that has uncertainty about its</td><td>any three

4(Expln)
+
3(Eg,)</td><td>7</td><td></td></tr>
</table>

occurrence is known as random signal. A random signal has irregular pattern and cannot be represented by the mathematical equations.

Thermal noise generated in an electric circuit is a common example of random signal.

(3) Periodic and non-periodic signal

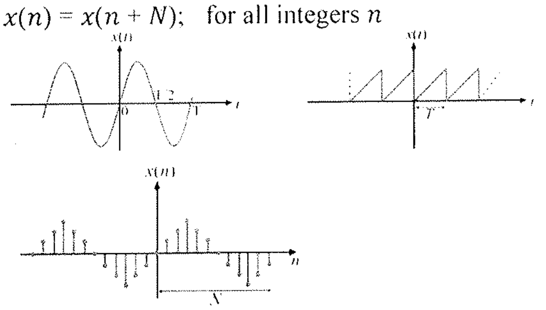
- Periodic Signals – A periodic signal is defined as a signal which has a definite pattern which repeats itself at regular intervals of time.

  A continuous time signal x(t) is said to be periodic if and only if

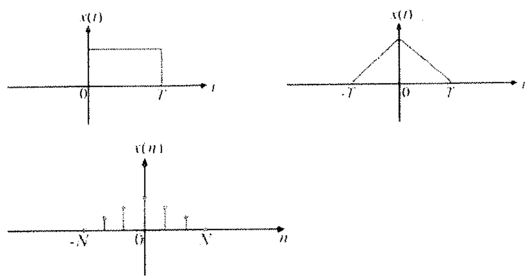
  $x(t + T) = x(t)$ for $-\infty < t < \infty$

  A discrete-time signal x(n) is said to be periodic if it satisfies the following condition

  $x(n) = x(n + N)$;   for all integers n



- Aperiodic Signals – A signal which does not repeat at regular intervals of time is known as aperiodic signal. The aperiodic signals are also called the non-periodic signals.



(4) Power and energy signals

Energy Signals – A signal is said to be an energy signal if and only if its total energy is finite ((i. e., $0 < E < \infty$).

The average power of an energy signal is zero, i.e., $P = 0$.

Examples of energy signals are aperiodic signals.

Power Signals – If a signal has finite average power (i. e., $0 < P < \infty$), it is called a power signal.

The total energy of a power signal is zero (i.e., $E = 0$).

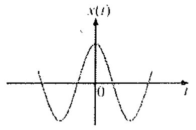
Periodic signals are the examples of power signals.

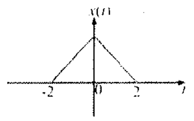
(5) Even and odd signal

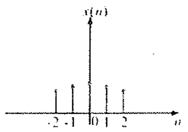
- A signal is said to be an even signal or symmetric signal if it satisfies the following condition:

  $x(n) = x(-n)$ or $x(t) = x(-t)$

  ex. Cosine signal, triangular signal



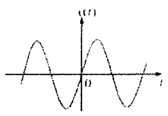


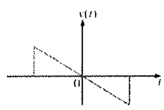


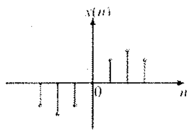
- A signal is said to be an odd signal or anti-symmetric signal if it satisfies the following condition

  $x(-n) = -x(n)$ or $x(-t) = -x(t)$.

  ex. Sine signal, signum function







(6) **Real and imaginary signals**

- A signal is said to be real when it satisfies the condition

  $x^*(t)=x(t)$

  or $x^*(n)=x(n)$

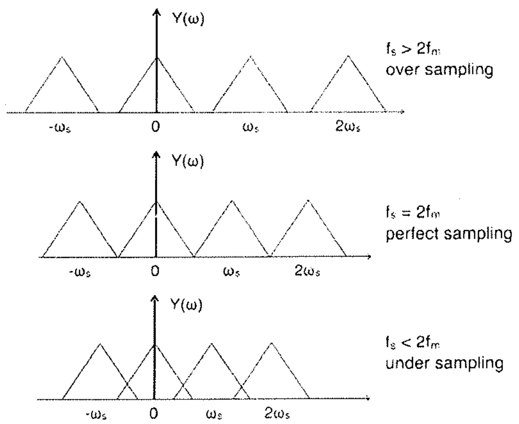
  For a real signal, imaginary part=0

- A signal is said to be imaginary if,

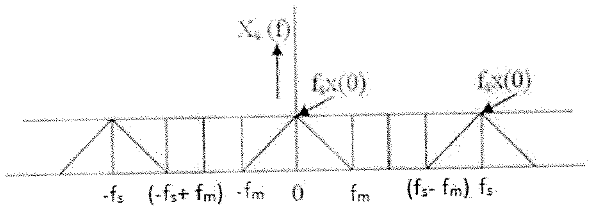
  $x^*(t)=-x(t)$

  or $x^*(n)=-x(n)$

  For an imaginary signal, real part=0

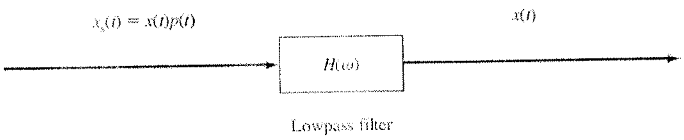
IX.	**Sampling Theorem** A bandlimited continuous time signal with maximum frequency f_m can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component f_m of message signal. i. e., $f_s \geq 2f_m$ Sampling of input signal $x(t)$ can be obtained by multiplying $x(t)$ with an impulse train $p(t)$ of period T_s. The sampled version of the continuous time signal is, $x_s(t) = x(t)p(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t-nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s)$ The spectrum of the sampled signal, $X_s(j\omega) = FT\{x_s(t)\}$ $= \frac{1}{T}\sum_{-\infty}^{+\infty} X(j(\omega-\omega_s)$ The Fourier transform of the sampled signal is thus given by an infinite sum of shifted replicas of the spectrum of the original signal  To reconstruct $x(t)$, we must recover input signal spectrum $X(\omega)$ from sampled signal spectrum $X_s(j\omega)$, which is possible when there is no overlapping between the cycles of $X_s(j\omega)$. For $f_s \geq 2f_m$, the signal is perfectly sampled without any information loss.	**2(theorem)** + **3(explntn)** + **2(diagrms)**	7	

But for $f_s < 2f_m$, over-lapping of spectrum occurs, which leads to mixing up and loss of information. This unwanted phenomenon of over-lapping is called as Aliasing

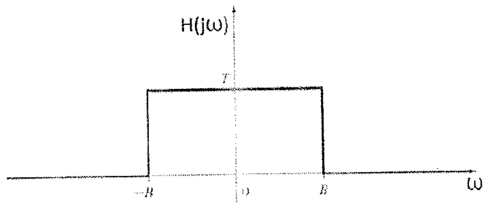


Signal reconstruction

In order to reconstruct the original signal $x(t)$, we can use an ideal lowpass filter on the sampled spectrum which has a bandwidth of any value between f_m and $(f_s - f_m)$. The filter will pass only the portion of sampled spectrum, $X_s(f)$, centred at $f = 0$ and will reject all its replicas at $f = nf_s$, for $n \neq 0$. This implies that the shape of the continuous time signal $x_s(t)$, will be retained at the output of the ideal filter.



Spectrum of LPF



$f_s<B<f_s-f_m$

X.	A continuous-time periodic signal can be decomposed into a sum of sinusoidal or complex exponential functions with different frequencies, amplitudes, and phases. This representation is known as the **Continuous Time Fourier series representation**. ie, The Fourier series allows us to express a periodic signal as a linear combination of harmonic components. A periodic signal is one which repeat itself periodically over $-\infty < t < \infty$ i.e., A signal is periodic if, $x(t) = x(t+T)$ for all t	**3(explntn) + 2(equation) + 2(eg)**	7

where, T is the fundamental period and $\omega_0 = 2\pi / T$ is referred to as the fundamental frequency.

The Fourier series is thus a decomposition of periodic signals into the sum of a (possibly infinite) number of complex exponentials whose frequencies are *harmonically* related.

The set of harmonically related complex exponentials is given by

$$\phi_k(t) = e^{jk\omega_0 t} = e^{jk(2\pi/T)t} \quad k = 0, \pm 1, +2, \ldots$$

Thus, a periodic signal $x(t)$ can be represented as

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \text{ where } \omega_0 = 2\pi / T = \text{Fundamental Frequency}$$

$c_n =$ fourier coefficients and $\pm n\omega_o =$ harmonic frequencies

The term for $n=0$ is a constant
The terms for $n=\pm 1$ are the fundamental/first harmonic components
The terms for $n=\pm 2$ are the second harmonic components
The terms for $n=\pm N$ are the N^{th} harmonic components

The frequency range of continuous time signal is from $-\infty$ to $+\infty$, and so CTFS has infinite frequency spectrum.

The Fourier coefficients can be evaluated using the equation,

$$c_n = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-jn\omega_0 t} dt$$

The Fourier coefficient c_n represents the amplitude and phase associated with the n^{th} frequency component. The Fourier coefficients provide the description of $x(t)$ in the frequency domain.

Example:

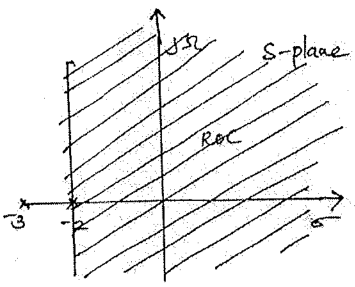
Consider the sinusoidal signal, $\sin(\pi t)$
The fundamental frequency of $\sin(\pi t)$ is π. By inspection, using Euler's theorem, we can write:

$$\sin(\pi t) = \frac{1}{2j} e^{j\pi t} - \frac{1}{2j} e^{-j\pi t}$$

So, $c_1 = \frac{1}{2j}$, $c_{-1} = -\frac{1}{2j}$ and $c_n = 0$ otherwise

Thus, here $\sin(\pi t)$ is expressed as a linear combination of Fourier coefficients, C_n

XI.		Any seven 1*7	7	
	1. Linearity $f_1(t) \xrightarrow{L.T.} F_1(s)$ with $ROC = R_1$ $f_2(t) \xrightarrow{L.T.} F_2(s)$ with $ROC = R_2$ $af_1(t) + bf_2(t) \xrightarrow{L.T.} aF_1(s) + bF_2(s)$: $ROC = R_1 \cap R_2$ 2. Time Shifting $f(t) \xrightarrow{L.T.} F(s)$ with $ROC = R$ $f(t - t_0) \xrightarrow{L.T.} e^{-st_0} F(s)$: $ROC = R$ 3. Time Scaling: $f\left(\frac{t}{a}\right) \xleftrightarrow{\mathcal{L}} aF(as)$ 4. Shift in S-domain $f(t) \xrightarrow{L.T.} F(s)$ with $ROC = R$ $e^{st_0}f(t)) \xrightarrow{L.T.} F(s - s_0)$: $ROC = R + Re\{s_0\}$ 5. Time-reversal $f(t) \xrightarrow{L.T.} F(s)$ with $ROC = R$ $f(-t) \xrightarrow{L.T.} F(-s)$ with $ROC = -R$ 6. Differentiation in S-domain $f(t) \xrightarrow{L.T.} F(s)$ with $ROC = R_1$ $tf(t) \xrightarrow{L.T.} \frac{-d}{ds} F(s)$: $ROC = R$ 7. Integration The integration theorem states that $\int_{0^-}^{t} f(\lambda)d\lambda \xleftrightarrow{\mathcal{L}} \frac{F(s)}{s}$ $\mathcal{L}\left(\int_{0^-}^{t} f(\lambda)d\lambda\right) = \frac{1}{s}F(s)$ 8. Convolution in Time if $f(t) \xrightarrow{L.T.} F(s)$ with $ROC = R_1$ and $h(t) \xrightarrow{L.T.} H(s)$with $ROC = R_2$ $f(t) * h(t) \xrightarrow{L.T.} F(s)H(s)$: $ROC = R_1 \cap R_2$ 9. Initial Value Theorem Initial value theorem is applied when in Laplace transform the degree of the numerator is less than the degree of the denominator $f(0) = \lim_{s \to \infty} sF(s)$			

	10. Final Value Theorem: If all the poles of sF(s) lie in the left half of the S-plane final value theorem is applied. $$f(\infty) = \lim_{s \to 0} sF(s)$$ 11. Multiplication by time: $$tf(t) \overset{\mathcal{L}}{\leftrightarrow} -\frac{dF(s)}{ds}$$ 12. Complex Shift: $$f(t)e^{-at} \overset{\mathcal{L}}{\longleftrightarrow} F(s+a)$$			
XII.	$x(t) = e^{-3t}u(t) + e^{-2t}u(t)$ $L\{e^{-3t}u(t)\} = \frac{1}{s+3}$ ROC(R_1): $Re(s) > -3$ $L\{e^{-2t}u(t)\} = \frac{1}{s+2}$ ROC (R_2): $Re(s) > -2$ Applying linearity property, $L\{x(t)\} = L\{e^{-3t}u(t)\} + L\{e^{-2t}u(t)\}$ $= \frac{1}{s+3} + \frac{1}{s+2}$ $= \frac{s+2+s+3}{(s+3)(s+2)} = \frac{2s+5}{(s+3)(s+2)}$ ROC of $x(t)$ is $R_1 \cap R_2$ ie, $Re(s) > -2$ 	**4(problem solving) + 3(fig)**	**7**	
XIII.	**(i) δ(t)** $X(s) = \int_0^{\infty} x(t)e^{-st}dt$ $= \int_0^{\infty} \delta(t)e^{-st}dt = 1$ $\Rightarrow \boxed{L[\delta(t)] = 1}$	**2+2+3**	**7**	

	(ii) u(t) $X(s)=\int_0^{\infty} x(t)e^{-st}dt=\int_0^{\infty} e^{-st}dt$ $=\frac{-1}{s}e^{-st}\Big\|_0^{\infty}=\frac{1}{s}$ $\boxed{L[u(t)]=\frac{1}{s}}$ **(iii) t u(t)** $r(t)=tu(t)$ $L[r(t)]=\int_0^{\infty} te^{-st}dt$ $=\frac{1}{s}te^{-st}\Big\|_0^{\infty}-\frac{1}{s^2}e^{-st}\Big\|_0^{\infty}=\frac{1}{s^2}$			
XIV.	$X(s)=\frac{s}{s^2+5s+6}$ $=\frac{s}{(s+2)(s+3)}$ $=\frac{A}{s+2}+\frac{B}{s+3}$ $A=\cancel{(s+2)}\frac{s}{\cancel{(s+2)}(s+3)}\Big\|_{s=-2}$ $=\frac{(-2)}{(-2+3)}=-2$ $B=\cancel{(s+3)}\frac{s}{(s+2)\cancel{(s+3)}}\Big\|_{s=-3}$ $=\frac{-3}{(-3+2)}=3$ $X(s)=\frac{-2}{(s+2)}+\frac{3}{s+3}$ Taking inverse Laplace transform we get $x(t)=-2e^{-2t}u(t)+3e^{-3t}u(t)$	**4(partial fraction solving) + 3(inverse LT)**	**7**	