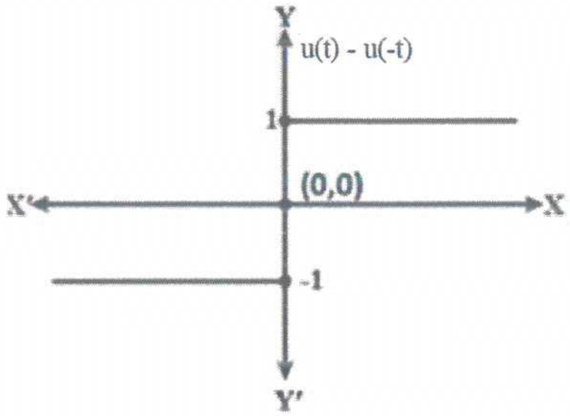
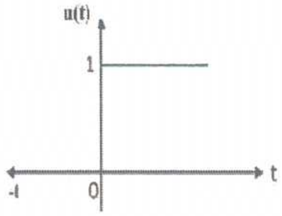
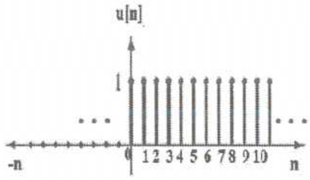
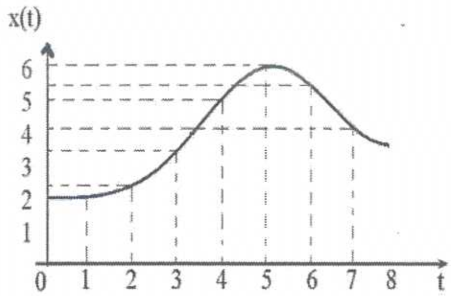
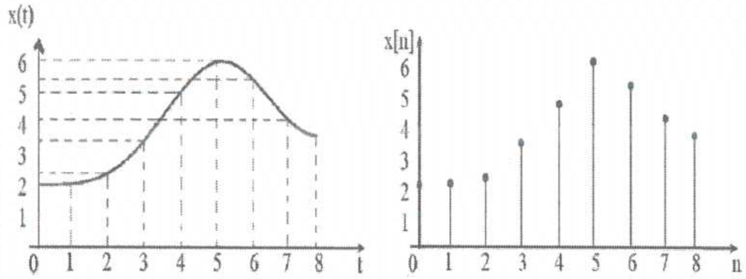
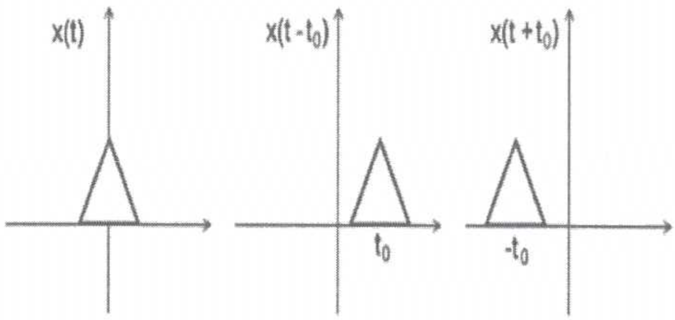
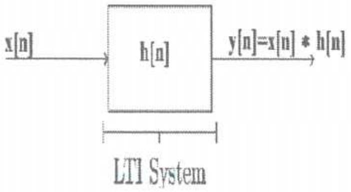
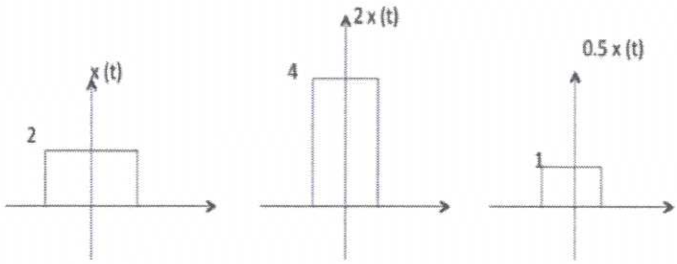
SCORING INDICATORS

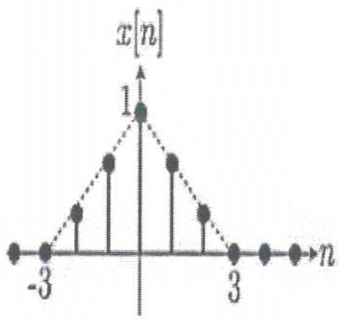
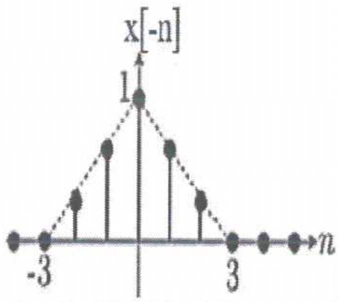
COURSE NAME : SIGNALS AND SYSTEMS
COURSE CODE : 5201 QID: 2109230110

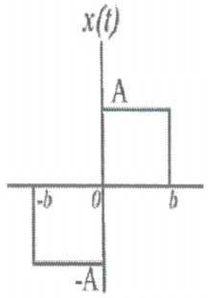
Q No	Scoring Indicators	Split score	Sub Total	Total score
	PART A			**9**
I. 1	1. Unit Step Function 2. Unit Impulse Function 3. Unit Ramp Function 4. Parabolic Function 5. Signum Function 6. Exponential Signal 7. Rectangular Function 8. Triangular Function 9. Sinusoidal Signal	Any 2 0.5 mark each	1	
I. 2		1	1	
I. 3	A system is said to be causal if its output depends upon present and past inputs, and does not depend upon future input. $y[n] = f(x[n], x[n-1], \ldots)$. All memory less systems are causal.	1	1	
I. 4	$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}, \quad a_k \,\&\, b_k \text{ are constants}$	1	1	
I. 5	Discrete-time Fourier transform	1	1	
I. 6	When the sampling frequency f_s is equal to twice the maximum frequency of the given signal, the sampling rate is called **Nyquist rate.** It is the minimum sampling frequency needed to reconstruct the analog signal from sampled waveform. The corresponding sampling interval $T_s = \frac{1}{2f_m}$ is called the Nyquist interval	1	1	

I. 7	**Continuous time** $u(t)=\begin{cases}1, & t>0\\ 0, & t<0\end{cases}$  **Discrete time** $u[n]=\begin{cases}1, & n\geq 0\\ 0, & n<0\end{cases}$  	Any 1	1	
I. 8	Time Scaling: $f\left(\frac{t}{a}\right) \xleftrightarrow{\mathcal{L}} aF(as)$	1	1	
I. 9	$\gamma(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s} = \Gamma(s)$	1	1	
	PART B			**24**
II. 1	**1. Continuous time signals:** ❖ These signals are defined over continuous independent variables. ❖ They are continuous in time and amplitude. ❖ Generally denoted by x(t). ❖ They are difficult to analyze, as they carry a huge number of values. In order to store these signals, we require an infinite memory.  	1.5 mark each	3	

	2. Discrete time signals: ❖ These signals are defined for only discrete values of time denoted by n. ❖ Discrete time signals are defined for only integer values of n. ❖ Obtained by sampling continuous time signals. ❖ $x[n] = x(t)\|_{t=nT}$, where T is the sampling time. ❖ They are continuous in amplitude and discrete in time and is denoted by x[n] 			
II. 2	**Time shifting** $x(t \pm t_0)$ is time shifted version of the signal x(t). $x(t + t_0) \rightarrow$ negative shift $x(t - t_0) \rightarrow$ positive shift 	3	3	
II. 3	The system is said to be stable only when the output is bounded for bounded input. For a bounded input, if the output is unbounded in the system, then it is said to be unstable. Example 1: $y(t) = x^2(t)$ Let the input is u(t) (unit step bounded input) then the output $y(t) = u^2(t) = u(t) =$ bounded output. Hence, the system is stable. Example 2: $y(t) = \int x(t)dt$ the input is u (t) (unit step bounded input) then the output $y(t) = \int u(t)dt =$ ramp signal (unbounded because amplitude of ramp is not finite it goes to infinite when $t \rightarrow$ infinite). Hence, the system is unstable.	1.5 mark each	3	

II. 4	**DISCRETE TIME SYSTEMS** A discrete-time system is a system that transform discrete-time input signals into discrete-time output signals  Here x[n] is the discrete-time input and y[n] is the discrete-time output. An example of discrete-time system is a simple model for the balance in a bank account from month-to-month. Discrete-time systems are described by difference equations.	Explanation - 2 Example – 1	3	
II. 5	$x[n] = 2.\ \delta[n+4] + 3.\ \delta[n+2] + 2\ .\delta[n+1] - 1.\ \delta[n] + 1\ \delta[n-4]$	3	3	
II. 6	Aliasing can be referred to as "the phenomenon of a high-frequency component in the spectrum of a signal, taking on the identity of a low-frequency component in the spectrum of its sampled version." The corrective measures taken to reduce the effect of Aliasing are: • A **low pass anti-aliasing filter** is employed, before the sampler, to eliminate the high frequency components, which are unwanted. • The signal which is sampled after filtering, is sampled at a rate slightly higher than the Nyquist rate. i.e., $f_s > 2f_m$	Definition -1 Measure - 2	3	
II. 7	**Amplitude scaling** Cx(t) is a amplitude scaled version of x(t) whose amplitude is scaled by a factor C.  	3	3	

II. 8	**Laplace Transform (F(s)):** The Laplace transform of a function $f(t)$ is defined as: $F(s) = \int_0^{\infty} e^{-st} f(t) dt$ In our case, $f(t) = 2u(t)$. Substituting this into the formula: $F(s) = \int_0^{\infty} e^{-st} \cdot 2u(t) dt$ Since $u(t)$ is equal to 1 for $t \geq 0$ and 0 for $t < 0$, we can simplify the integral: $F(s) = \int_0^{\infty} 2e^{-st} dt$ Now, we can solve this integral. The Laplace transform of 2 is simply $\frac{2}{s}$. $F(s) = \frac{2}{s}$ Now, we can solve this integral. The Laplace transform of 2 is simply $\frac{2}{s}$. $F(s) = \frac{2}{s}$ 2. **Region of Convergence (ROC):** The ROC for a given Laplace transform is the set of values of s for which the integral converges. In our case, the Laplace transform $F(s)$ is defined for all values of s. Therefore, the Region of Convergence (ROC) for the Laplace transform of $2u(t)$ is the entire complex plane, which means there are no restrictions on s. That's it! We have found the Laplace transform and the Region of Convergence (ROC) for the given simple function.	3	3	
II.9	A signal is said to be even when it satisfies the condition **x(t) = x(-t) or x[n] = x[-n]** 	1.5 marks each	3	

	A signal is said to be odd when it satisfies the condition **x(t) = -x(-t) or x[n] = - x[-n]**  			
II.10	In this case, $F(s) = \frac{1}{s}$, so we want to find $f(t)$ such that: $f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$ To find the inverse Laplace transform, we can use a well-known result: $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$ where $u(t)$ is the unit step function, defined as: $u(t) = \begin{cases} 1, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0 \end{cases}$ So, the inverse Laplace transform of $\frac{1}{s}$ is: $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$ This means that the inverse Laplace transform of $\frac{1}{s}$ is the unit step function $u(t)$.	3	3	
	PART C			**42**
III.	a) A function of one or more independent variables which contains some information is called a signal.	1 mark	7	

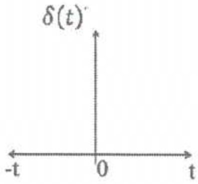
b)

2. Unit Impulse Function

Continuous time

Continuous time unit impulse function is also called Dirac delta function. It is only defined at t=0.

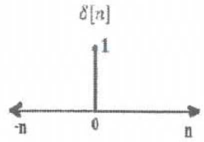
$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



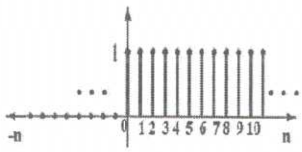
Discrete time

Discrete time unit impulse function is also called unit sample sequence. It is defined only at n=0.

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



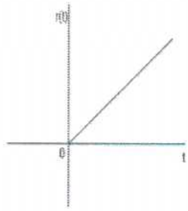
3. Unit Ramp Function

Amplitude increases linearly with time. Unit ramp function has unit slope.

Continuous time

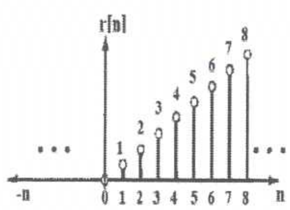
$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$r(t)$ =t.u(t)



Discrete time

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

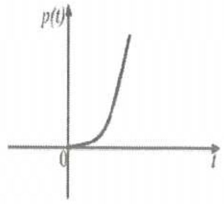


Any 6
1 mark each

4. **Parabolic Function**

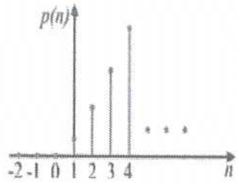
Continuous time

$$p(t) = \begin{cases} \frac{t^2}{2}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Discrete time

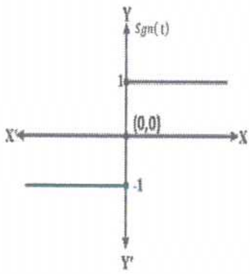
$$p[n] = \begin{cases} \frac{n^2}{2}, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



5. **Signum Function**

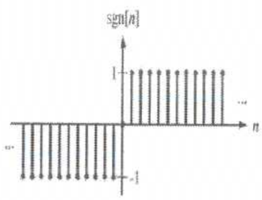
Continuous time

$$Sgn(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$$



Discrete time

$$\operatorname{sgn}[n] = \begin{cases} 1, & n > 0 \\ 0, & n = 0 \\ -1, & n < 0 \end{cases}$$



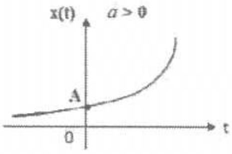
6. **Exponential Signal**

Continuous time

$$x(t) = Ae^{at}$$

'A' and 'a' are real numbers. 'A' is the amplitude of the exponential signal measured at t= 0.
'a' can be either positive or negative.
At t = $-\alpha$, $x(t) = Ae^{at}$ becomes $x(t) = Ae^{a.-\alpha}$ or $x(t) = Ae^{-\alpha}$ = 0
At t = 0, $x(t) = Ae^{at}$ becomes $x(t) = Ae^{a.0}$ or $x(t) = Ae^{0}$ = 1
At t = α, $x(t) = Ae^{at}$ becomes $x(t) = Ae^{a.\alpha}$ or $x(t) = Ae^{\alpha}$ = α

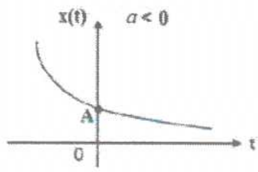
A. Rising when 'a' is positive



$$x(t) = \begin{cases} 0 \; at \; t = -\alpha : Ae^{-\alpha} = 0 \\ 1 \; at \; t = 0 : Ae^{0} = 1 \\ \alpha \; at \; t = \alpha : Ae^{\alpha} = \alpha \end{cases}$$

B. Decaying when 'a' is negative

B. Decaying when 'a' is negative

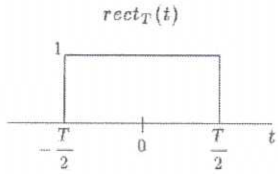


$$x(t) = \begin{cases} \alpha \; at \; t = -\alpha : Ae^{\alpha} = \alpha \\ 1 \; at \; t = 0 : Ae^{0} = 1 \\ 0 \; at \; t = \alpha : Ae^{-\alpha} = 0 \end{cases}$$

7. Rectangular Function

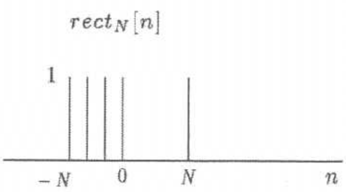
Continuous time

$$rect(t) = \begin{cases} 1 & \text{if } \frac{-1}{2} \geq t \geq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



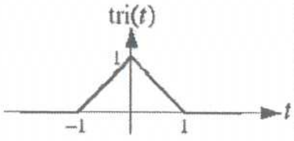
Discrete time

$$rect[n] = \begin{cases} 1 & \text{if } \frac{-1}{2} \geq n \geq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



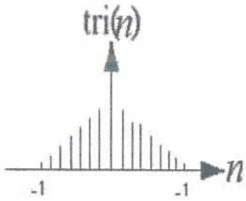
8. **Triangular Function**
   Continuous time

$$\mathrm{tri}(t)=\begin{cases}1-|t| & , |t|<1\\ 0 & , |t|\geq 1\end{cases}$$



Discrete time

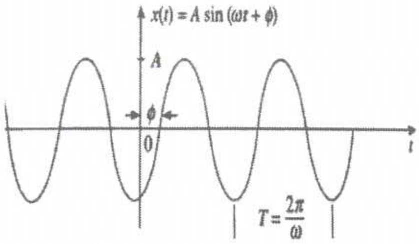
$$\mathrm{tri}[n]=\begin{cases}1-|n| & , |n|<1\\ 0 & , |n|\geq 1\end{cases}$$



9. **Sinusoidal Signal**

Continuous time

x(t)=A sin (ωt+ϕ)



x(t) = independent variables
A = amplitude
ω = angular frequency in radians
$T = \frac{2\pi}{\omega}$
ϕ = phase angle in radians

IV

No of sample m = 4
No of sample n = 4
No of sample in y[n] = m + n -1 = 4 + 4 – 1 = 7

Sum by column method

$$\begin{array}{lccccccc}
x(n) = & 1 & 2 & 3 & 4 & & & \\
h(n) = & 1 & 2 & 1 & 2 & & & \\
\hline
 & 1 & 2 & 3 & 4 & & & \\
 & & 2 & 4 & 6 & 8 & & \\
 & & & 1 & 2 & 3 & 4 & \\
 & & & & 2 & 4 & 6 & 8 \\
\hline
y(n) = & 1 & 4 & 8 & 14 & 15 & 10 & 8
\end{array}$$

Now define a new signal $\tilde{x}(t)$,which is a periodic extension of x(t) with period T. In other words, $\tilde{x}(t)$ is obtained by repeating x(t), where each copy is shifted T units in time. This $\tilde{x}(t)$ has a Fourier series representation, which we found in the last section to be

$$a_0 = \frac{2T_1}{T}, \quad a_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T},$$

Now recall that the Fourier series coefficients are calculated as follows:

$$a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \tilde{x}(t)e^{-jk\omega_0 t}dt.$$

However, we note that x(t) = $\tilde{x}(t)$ in the interval of integration, and thus

$$a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-jk\omega_0 t}dt.$$

Furthermore, since x(t) is zero for all t outside the interval of integration, we can expand the limits of the integral to obtain

V

$$a_k = \frac{1}{T}\int_{-\infty}^{\infty} x(t)e^{-jk\omega_0 t}dt.$$

Let us define

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt.$$

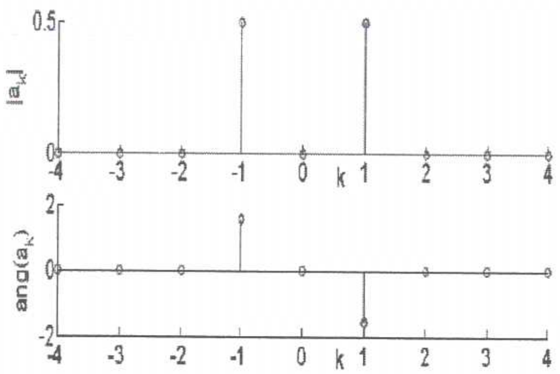
This is called the Fourier transform of the signal x(t), and the Fourier series coefficients can be viewed as samples of the Fourier transform, scaled by T_1, i.e.,

$$a_k = \frac{1}{T}X(jk\omega_0), k \in \mathbb{Z}.$$

Now consider the fact that

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(jk\omega_0)e^{jk\omega_0 t}$$

Since $\omega_0 = \frac{2\pi}{T}$, this becomes

	$$\tilde{x}(t) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} X(jk\omega_0)e^{jk\omega_0 t}\omega_0$$ Now consider what happens as the period T gets bigger. In this case, $\tilde{x}(t)$ approaches x(t), and so the above expression becomes a representation of x(t). As $T \to \infty$, we have $\omega_0 \to 0$. Since each term in the summand can be viewed as the area of the rectangle whose height is $X(jk\omega_0)e^{-jk\omega_0 t}$ and whose base goes from $k\omega_0$ to $(k+1)\omega_0$, we see that as $\omega_0 \to 0$, the sum on the right-hand side approaches the area underneath the curve $X(j\omega)e^{-j\omega t}$ (where t is held fixed). Thus, as $T \to \infty$, we have $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$ Thus, we have the following Given a continuous-time signal x(t), the **Fourier Transform** of the signal is given by $$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$			
VI.	The fundamental period of $\sin(\omega_0 t)$ is ω_0. By inspection we can write: $$\sin(\omega_0 t) = \frac{1}{2j}e^{j\omega_0 t} - \frac{1}{2j}e^{-j\omega_0 t}$$ So, $c_1 = \frac{1}{2j}$, $c_{-1} = -\frac{1}{2j}$ and $c_k = 0$ otherwise  	Explanation - 4 mark Diagram – 3 mark	7	

VII	Time Variant and Time Invariant Systems A system is said to be time invariant if the behaviour and characteristics of the system do not change with time. Thus, a system is said to be time invariant if a time delay or time advance in the input signal leads to identical delay or advance in the output signal. Mathematically if $\{y[n]\} = T\{x[n]\}$ Then $\{y[n-n_0]\} = T(\{x[n-n_0]\})$ The condition for time invariant system is: y(n, t) = y(n-t) The condition for time variant system is: y (n, t) ≠ y(n-t) where y (n, t) = T[x(n-t)] = input change y(n-t) = output change Example 1: y(n) = x(-n) y(n, t) = T[x(n-t)] = x(-n-t) y(n-t) = x(-(n-t)) = x(-n + t) ∴ y(n, t) ≠ y(n-t). Hence, the system is time variant. Example 2: y(n)=nx(n) y(n, t)= T[x(n-t)]=nx(n-t) y(n-t)=(n-t)x(n-t) ∴ y(n, t) ≠ y(n-t). Hence, the system is time variant.	Explanation - 4 mark Example – 3 mark	7	
VIII	Static and Dynamic Systems Static system is memory-less whereas dynamic system is a memory system. A system is said to be memory less if the output for each value of the independent variable at a given time n depends only on the input value at time n. For example, system specified by the relationship y[n] = cos(x[n]) + z is memory less. A particularly simple memory less system is the identity system defined by y[n] = x[n] In general we can write input-output relationship for memory less system as y[n] = g(x[n]). Not all systems are memory less. A simple example of system with memory is a delay defined by y[n] = x[n − 1]. A system with memory retains or stores information about input values at times other than the current input value. Example 1: y(t) = 2x(t) For present value t=0, the system output is y(0) = 2x(0). Here, the output is only dependent upon present input. Hence the system is memory less or static. Example 2: y(t) = 2 x(t) + 3 x(t-3) For present value t=0, the system output is y(0) = 2x(0) + 3x(-3). Here x(-3) is past value for the present input for which the system requires memory to get this output. Hence, the system is a dynamic system.	1 mark each	7	

IX	**1. Linearity** If $x(n) \xleftrightarrow{DTFS} c_k$ and $y(n) \xleftrightarrow{DTFS} d_k$ Then, $ax(n)+by(n) \xleftrightarrow{DTFS} ac_k + bd_k$ i.e., Fourier Series is a linear operation. **2. Time Shifting** If $x(n) \xleftrightarrow{DTFS} c_k$ Then according to time shifting property, $x(n-m) \xleftrightarrow{DTFS} e^{-jk\omega_0 m} c_k$ i.e., Magnitude of Fourier Series coefficients remains unchanged when the signal is shifted in time. **3. Frequency Shifting** If $x(n) \xleftrightarrow{DTFS} c_k$ Then according to frequency shifting property, $e^{jm\omega_0 n} x(t) \xleftrightarrow{DTFS} c_{k-m}$ **4. Time Scaling** If $x(n)$ is periodic with period N, then $x(n/m)$ (where N multiple of m) will be periodic with period mN If $x(n) \xleftrightarrow{DTFS} c_k$ Then $x\left(\frac{n}{m}\right) \xleftrightarrow{FS} \frac{1}{m} c_k$ **5. Time Reversal** Time inversion property states that If $x(n) \xleftrightarrow{DTFS} c_k$ Then, $x(-n) \xleftrightarrow{DTFS} c_{-k}$ **6. Multiplication** If $x(n) \xleftrightarrow{DTFS} c_k$ and $y(n) \xleftrightarrow{DTFS} d_k$ We have $x(n)y(n) \xleftrightarrow{DTFS} \sum_{m=0}^{N-1} c_m d_{k-m}$ Multiplication in time domain leads to convolution in Fourier series domain. **7. Convolution** If $x(n) \xleftrightarrow{DTFS} c_k$ and $y(n) \xleftrightarrow{DTFS} d_k$ We have $\sum_{m=0}^{N-1} x(m)y((n-m))_N \xleftrightarrow{DTFS} Nc_k d_k$ **8. Symmetry** Symmetry properties state that If $x(n)$ is real, then $c_k = c_{-k}^*$ If $x(n)$ is imaginary, then $c_k = -c_{-k}^*$	Any 7 1 mark each	7	

X	Let's compute the Fourier transform step by step for both terms in the given signal: 1. $e^{-at}u(t)$: First, consider the term $e^{-at}u(t)$. This represents a right-sided exponential decay with a time constant a. The Fourier transform of $e^{-at}u(t)$ can be found directly using the formula for the Fourier transform of an exponential function: $\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{j2\pi f+a}$ 2. $e^{-at}u(-t)$: Now, consider the term $e^{-at}u(-t)$. This represents a left-sided exponential decay with a time constant a. To find its Fourier transform, you can use a similar approach. First, introduce a substitution by letting $t' = -t$, which reverses the time axis: $e^{-at}u(-t) = e^{at'}u(t')$ Now, apply the Fourier transform to this expression: $\mathcal{F}\{e^{-at}u(-t)\} = \mathcal{F}\{e^{at'}u(t')\}$ Use the Fourier transform property of time reversal: $\mathcal{F}\{e^{at'}u(t')\} = \frac{1}{j2\pi f-a}$ 3. **Combining Both Terms:** Now, combine the Fourier transforms of both terms: $X(f) = \frac{1}{j2\pi f+a} - \frac{1}{j2\pi f-a}$ To simplify this expression, you can find a common denominator: $X(f) = \frac{-2a}{(2\pi f)^2+a^2}$ So, the Fourier transform of the given signal $x(t) = e^{-at}u(t) - e^{-at}u(-t)$ is: $X(f) = \frac{-2a}{(2\pi f)^2+a^2}$	7	7	

XI	**PROPERTIES OF LAPLACE TRANSFORM** 1. Linearity $f_1(t) \xrightarrow{L.T.} F_1(s)$ with $ROC = R_1$ $f_2(t) \xrightarrow{L.T.} F_2(s)$ with $ROC = R_2$ $af_1(t) + bf_2(t) \xrightarrow{L.T.} aF_1(s) + bF_2(s)$: $ROC = R_1 \cap R_2$ 2. Time Shifting $f(t) \xrightarrow{L.T.} F(s)$ with $ROC = R$ $f(t-t_0) \xrightarrow{L.T.} e^{-st_0} F(s)$: $ROC = R$ 3. Time Scaling: $f\left(\frac{t}{a}\right) \xleftrightarrow{\mathcal{L}} aF(as)$ 4. Shift in S-domain $f(t) \xrightarrow{L.T.} F(s)$ with $ROC = R$ $e^{st_0} f(t)) \xrightarrow{L.T.} F(s-s_0)$: $ROC = R + Re\{s_0\}$ 5. Time-reversal $f(t) \xrightarrow{L.T.} F(s)$ with $ROC = R$ $f(-t) \xrightarrow{L.T.} F(-s)$ with $ROC = -R$ 6. Differentiation in S-domain $f(t) \xrightarrow{L.T.} F(s)$ with $ROC = R_1$ $tf(t) \xrightarrow{L.T.} \frac{-d}{ds} F(s)$: $ROC = R$ The differentiation property of the Laplace Transform. We will use the differentiation property widely. It is repeated below (for first, second and n^{th} order derivatives)	Any 7 1 mark each	7	

$$\frac{df(t)}{dt} \xleftrightarrow{\mathcal{L}} sF(s) - f(0^-)$$

$$\frac{d^2f(t)}{dt^2} \xleftrightarrow{\mathcal{L}} s^2F(s) - sf(0^-) - \dot{f}(0^-)$$

$$\frac{d^nf(t)}{dt^n} \xleftrightarrow{\mathcal{L}} s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\dot{f}(0^-) - \cdots - s\,f^{(n-2)}(0^-) - f^{(n-1)}(0^-)$$

7. Integration

The integration theorem states that

$$\int_{0^-}^{t} f(\lambda)d\lambda \xleftrightarrow{\mathcal{L}} \frac{F(s)}{s}$$

$$\mathcal{L}\left(\int_{0^-}^{t} f(\lambda)d\lambda\right) = \frac{1}{s}F(s)$$

8. Convolution in Time

$$\text{if } f(t) \xrightarrow{L.T.} F(s) \text{ with } ROC = R_1 \text{ and } h(t) \xrightarrow{L.T.} H(s) \text{ with } ROC = R_2$$

$$f(t) * h(t) \xrightarrow{L.T.} F(s)H(s): ROC = R_1 \cap R_2$$

9. Initial Value Theorem

Initial value theorem is applied when in Laplace transform the degree of the numerator is less than the degree of the denominator

$$f(0) = \lim_{s\to\infty} sF(s)$$

10. Final Value Theorem:

If all the poles of sF(s) lie in the left half of the S-plane final value theorem is applied.

$$f(\infty) = \lim_{s\to 0} sF(s)$$

11. Multiplication by time:

$$tf(t) \overset{\mathcal{L}}{\leftrightarrow} -\frac{dF(s)}{ds}$$

12. Complex Shift:

$$f(t)e^{-at} \xleftrightarrow{\mathcal{L}} F(s+a)$$

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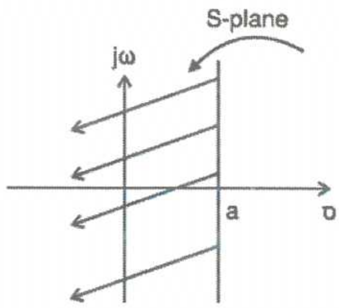
a)

$L[x(t)] = L[e^{at}u(-t)] = \frac{1}{s-a}$

ROC: Re{s} < a



b)

$F(s) = \int_0^{\infty} e^{-st}\cos(3t)\cdot e^{-2t}u(t)dt$

Simplifying the exponent terms:

$F(s) = \int_0^{\infty} e^{-(s+2)t}\cos(3t)u(t)dt$

Now, we can solve this integral. The Laplace transform of $\cos(at)u(t)$ is $\frac{s}{s^2+a^2}$ for $s > 0$.

In our case, $a = 3$ and $s + 2 > 0$ for the ROC, so we have:

$F(s) = \frac{s}{(s+2)^2+3^2}$

2. **Region of Convergence (ROC):**

The ROC for a given Laplace transform is the set of values of s for which the integral converges. In our case, the ROC is defined by $s + 2 > 0$, which simplifies to $s > -2$. Therefore, the Region of Convergence (ROC) for the Laplace transform of $\cos(3t)\cdot e^{-2t}u(t)$ is $s > -2$.

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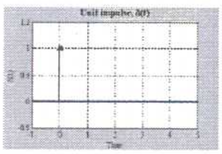
The Unit Impulse

The impulse function is everywhere but at t=0, where it is infinitely large. The area of the impulse function is one. The impulse function is drawn as an arrow whose height is equal to its area.

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \text{undefined}, & t = 0 \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(t)dt = 1$$



To find the Laplace Transform, we apply the definition

$$\Delta(s) = \int_{0^-}^{\infty} \delta(t)e^{-st}dt$$

Now we apply the shifting property of the impulse. Since the impulse is 0 everywhere but t=0, we can change the upper limit of the integral to 0^+.

$$\Delta(s) = \int_{0^-}^{0^+} \delta(t)e^{-st}dt$$

Since e^{-st} is continuous at t=0, that is the same as saying it is constant from $t=0^-$ to $t=0^+$. So, we can replace e^{-st} by its value evaluated at t=0.

$$e^{-st}\big|_{t=0} = e^{-s0} = 1$$

$$\Delta(s) = \int_{0^-}^{0^+} \delta(t)\cdot 1\cdot dt = 1$$

$$\delta(t) \xleftrightarrow{\mathcal{L}} 1$$

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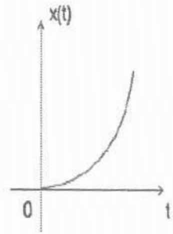
. **The Parabolic**

A unit parabolic function is defined as

$y(t) = t^2/2$; for $t > 0$

$= 0$; elsewhere



(2 marks)

$Y(S) = {}_0\int^{\infty} y(t).\, e^{-st}.dt.$

$= {}_0\int^{\infty} t^2/2.\, e^{-st}.dt.$

$= [t^2/2. e^{-st}/-s]_0^{\infty} - {}_0\int^{\infty} t.e^{-st}.(1/-s).dt.$

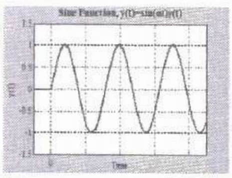
$= 1/s[{}_0\int^{\infty} t.e^{-st}dt.]$

$= 1/s\ .1/s^2$

$= 1/s^3$

. **The Sine**

$$y(t) = \sin(\omega t)\gamma(t)$$



As before, start with the definition of the Laplace transform

$$Y(s) = \int_{0^-}^{\infty} \sin(\omega t) e^{-st} dt$$

Here it becomes useful to use Euler's identity for the sine

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$Y(s) = \int_{0^-}^{\infty} \frac{e^{j\omega t} - e^{-j\omega t}}{2j} e^{-st} dt = \frac{1}{2j}\int_{0^-}^{\infty} e^{j\omega t} e^{-st} dt - \frac{1}{2j}\int_{0^-}^{\infty} e^{-j\omega t} e^{-st} dt$$

(3 marks)

But we've already done this integral (the exponential function, above)

$$Y(s) = \frac{1}{2j}\frac{1}{s - j\omega} - \frac{1}{2j}\frac{1}{s + j\omega}$$

Let's put this over a common denominator

$$Y(s) = \frac{1}{2j}\frac{1}{(s - j\omega)}\frac{(s + j\omega)}{(s + j\omega)} - \frac{1}{2j}\frac{1}{(s + j\omega)}\frac{(s - j\omega)}{(s - j\omega)}$$

$$= \frac{1}{2j}\frac{(s + j\omega) - (s - j\omega)}{\left(s^2 - \cancel{sj\omega} + \cancel{sj\omega} - (j\omega)^2\right)} = \frac{1}{2j}\frac{2j\omega}{s^2 + \omega^2}$$

$$= \frac{\omega}{s^2 + \omega^2}$$

$$\sin(\omega t) \xleftrightarrow{\mathcal{L}} \frac{\omega}{s^2 + \omega^2}$$

XIV	a) $F(s) = \frac{2}{s^2+4}$ $L^{-1}\left[\frac{2}{s^2+4}\right] = L^{-1}\left[\frac{2}{s^2+2^2}\right] = sin2t$ b) $F(s) = \frac{s+1}{s^2+2s+10}$ $L^{-1}\left[\frac{s+1}{s^2+2s+10}\right] = L^{-1}\left[\frac{s+1}{(s+1)^2+9}\right] = L^{-1}\left[\frac{s+1}{(s+1)^2+3^2}\right] = e^{-t}cos3t$	2 marks 5 marks	7	