ANSWER KEY

STRENGTH OF MATERIALS-SET B

SUBJECT CODE : 3021 **QID: 2110220246**

PART A

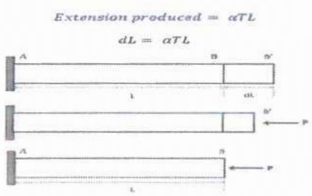
Answer all the following questions **(9 x 1 = 9 Marks)**

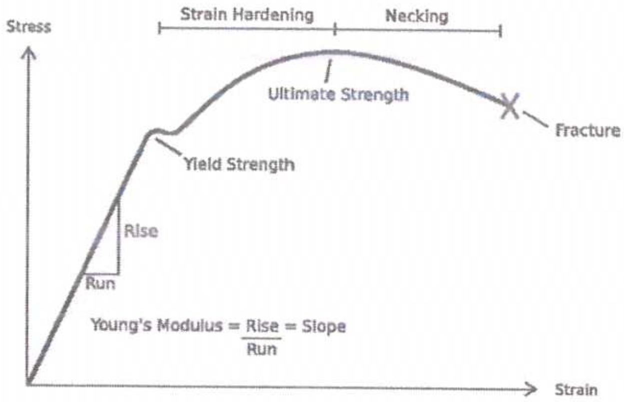
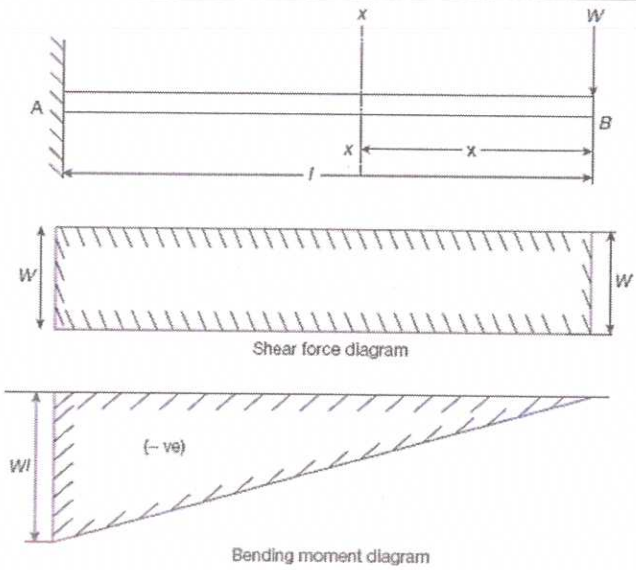
I

Q.No	Answer	Split up	Total Mark
1	Ratio of lateral strain to linear strain	1	1
2	A factor of safety is the load-carrying capacity of a system beyond what the system actually supports	1	1
3	Fixed beam	1	1
4	A shear force diagram is one which shows variation in shear force along the length of the beam	1	1
5	Roller support	1	1
6	Beam extends beyond one support or both	1	1
7	The slenderness ratio is defined as the ratio of length l to the radius of gyration k, represented as l/k.	1	1
8	a) longitudinal stress b) hoop stress	1	1
9	a) bending spring b) torsion spring c) helical spring – open coil/closed coil	1	1

II.

PART B

Q.No	Answer	Split up	Total Mark
1	Thermal stress is the stress produced by any change in the temperature of the material. Thermal stress is induced in a body when the temperature of the body is raised or lowered and the body is not allowed to expand or contract freely. Thermal stress includes both heat and cold stress 	1 1 1	3

2		2 1	3
3	Types of forces a) Tensile force b) Compressive force c) Shear force	1 1 1	3
4	Types of stress a) Tensile stress b) Compressive stress c) Shear stress	1 1 1	3
5	The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force. It is briefly written as S.F. The algebraic sum of the moments of all the forces acting to the right or left of the section is known as bending moment		3
6		1.5 1.5	3

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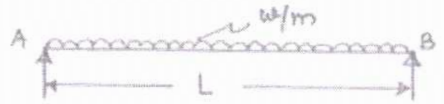
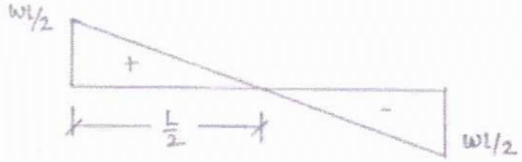
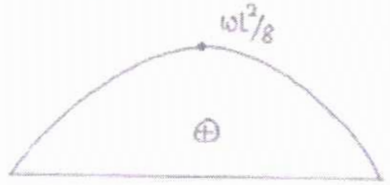
A shear force diagram is one which shows variation in shear force along the length of the beam. Bending moment may be defined as "the sum of moments about that section of all external forces acting to one side of that section".





Shear force diagram.



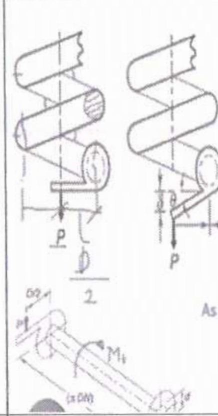
Bending moment diagram.

8

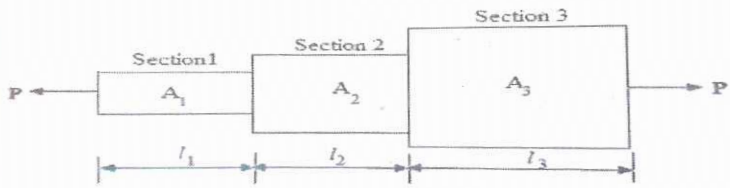
End conditions

3 3

S.No.	End conditions of column	Crippling load in terms of Actual length	Crippling load in terms of Effective length	Relation between effective length and actual length
1.	Both ends hinged	$\frac{\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{L^2}$	$L = l$
2.	One end is fixed and other is free	$\frac{\pi^2 EI}{4l^2}$	$\frac{\pi^2 EI}{L^2}$	$L = 2l$
3.	Both ends fixed	$\frac{4\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{L^2}$	$L = \frac{l}{2}$
4.	One end fixed and other is hinged	$\frac{2\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{L^2}$	$L = \frac{l}{\sqrt{2}}$

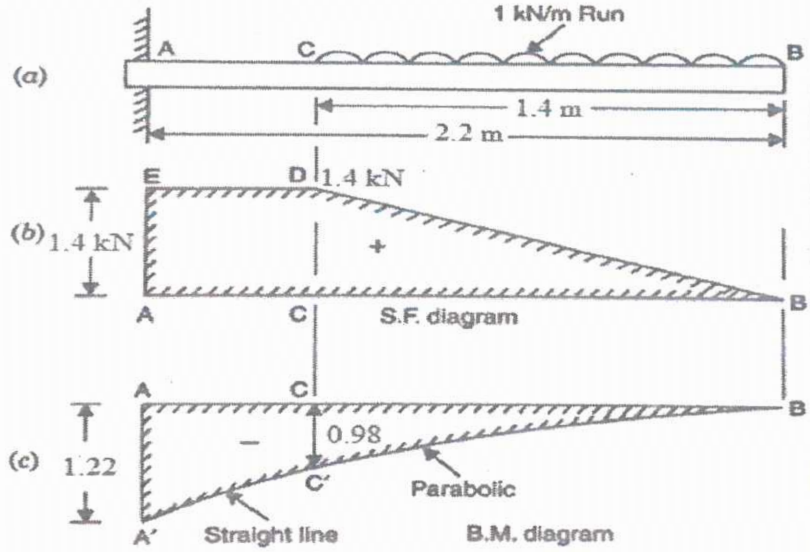
9	 The angle of twist (θ) for the equivalent bar, illustrated in Fig. (b), is given by, $\frac{M_t}{J}=\frac{G\theta}{l}$ $\theta=\frac{M_t l}{J G}$ ....(1) where, θ = angle of twist (radians) G = modulus of rigidity M_t = torsional moment =(PD/2) l = length of bar = (πDN) J = polar moment of inertia of bar = $\frac{\pi d^4}{32}$ Substituting values in Eq. (1), $\theta=\frac{(PD/2)(\pi DN)}{\frac{\pi d^4}{32}G}$ $\theta=\frac{16PD^2N}{Gd^4}$ ....(2) As shown in Fig. , the axial deflection 'δ' of the spring, for small values of θ, is given by, δ = θ × (length of bracket)	3	3
10	Assumptions in torsional equation 1. Material of shaft uniform 2. Shaft is circular cross section 3. Twist along the length of shaft is uniform 4. Distance b/w any two normal c/s remains same	1	

PART C

III	**3.Bars of varying cross section**  $\delta l = \delta l_1 + \delta l_2 + \delta l_3 = \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E} + \frac{P l_3}{A_3 E}$ $= \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$ = 796mm	3 4	7

	OR		
IV	Given $\alpha = 12\times10^{-6}/^0C$ $E = 2 \times 10^5 N/mm^2$ $l = 25m = 25\times10^3 mm$ $t_1 = 20^0C$ $t_2 = 65^0C$ $t = 65^0C - 20^0C = 45^0C$ $\delta = 6 mm$	1	
	Free expansion, $\delta l = l\,\alpha\,t = 25\times10^3 \times 12\times10^{-6} \times 45 = 13.5$ mm Thermal stress (do not yield), σ $= \alpha t E = 12\times10^{-6} \times 45 \times 2\times10^5 = 108$ N/mm².	2	7
	Thermal stress(yield by 6mm) σ $= \left(\alpha t - \dfrac{\delta}{l}\right)E$	1	
	$= \left(12\times10^{-6}\times45 - \dfrac{13.5}{25\times10^3}\right)\times2\times10^5 = 60 N/mm^2$.	2	

V



2

- Shear force

$SF_X = W$ 2

$F_x = w \times x$ 1

$= 1.0 \times x$

At B , $x = 0$ hence $F_B = 0$

At C, $x = 1.4$ hence $F_C = 1.0 \times 1.4 = 1.4$ kN

2 7

- Bending moment

$$M_x = -(w \times x)\frac{x}{2} = -\left(1.\frac{x^2}{2}\right) = -\frac{x^2}{2} \quad \text{.....................(i)}$$

(The bending moment is negative as for the right portion of the section the moment of load :
x is clockwise)

At B, x = 0 hence $M_B = -\frac{0^2}{2} = 0$

At C, x = 1.4 hence $M_C = -\frac{1.4^2}{2} = -0.98$ Nm

= (Load due to UDL) × (x − 0.7)

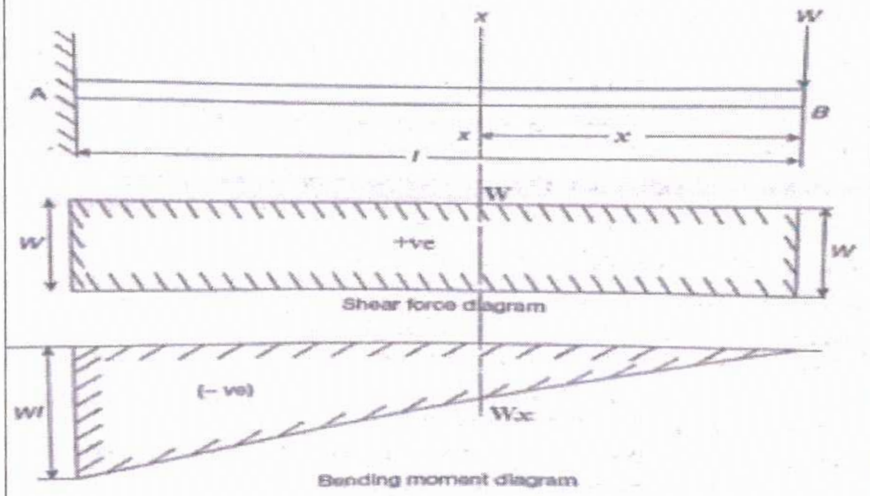
Mx = − 1.4 × (x − 0.7) ....(ii)

(− ve sign is due to clockwise moment for the right portion]

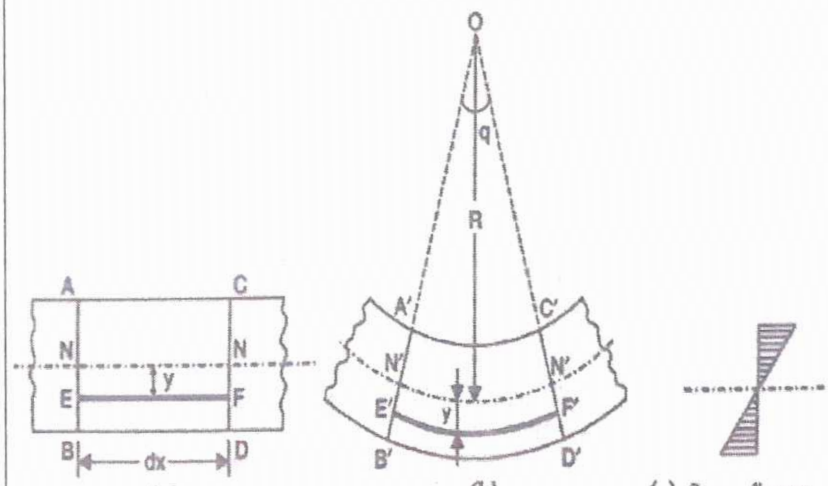
From the equation (ii) the bending moment follows a straight line law between A and C.

At C, x = 1.4 m hence $M_C = -1.4(1.4 - 0.7) = -\ 0.98$ Nm

At A, x − 2.2 m hence $M_A -\ 1.4(2.2 - 0.7) = -\ 1.22$ Nm

VI		2	7
		5	
	• Support reactions $R_A = W$ • Shear force $SF_X = W$ • Bending moment BM at section x $BM_X = -W x$ at $x=0$, $BM_A=0$ at $x=l$, $BM_A = -Wl$		
		1	
		1	

VIII	Given Width of the beam, b = 150 mm Depth of the beam, d = 300 mm ∴ Moment of inertia, $I = \frac{bd^3}{12} = \frac{150\times300^3}{12} = 3.375\times10^8\ mm^4$. Length of the beam, l = 4m = 4×10^3mm Maximum bending stress, σ_b = 8 N/mm2. E = $0.1\times10^5\ N/mm^2$	3	
	We know that Moment of resistance, $M = \sigma Z$ Where $Z = \frac{bd^2}{6} = \frac{150\times300^2}{6} = 2250000\ mm^3$. ∴ M $= 8\times2250000 = 18\times10^6$ N-mm We also know that Bending moment due to central point load, M $= \frac{Wl}{4} = 18\times10^6$ ∴ W $= \frac{18\times10^6\times4}{4\times10^3} = 18\times10^3$N	4	7
	Maximum deflection, $y = \frac{Wl^3}{48EI} = \frac{18\times10^3\times(4\times10^3)^3}{48\times0.1\times10^5\times3.375\times10^8} = 7.11\ mm$ **(Ans)**		

IX	**Theory of simple bending**		
	(a) (b) (c) Stress diagram	4	
	When a beam is loaded, it is bent and it is subjected to bending moment and shear force which vary from section to section. Therefore, the beam section develops stresses to resist these bending moments and shear forces namely bending stress and shear stress respectively. These stresses are determined independently. Here, we are neglecting the effect of shear and considering the effect of moment alone to find the stresses due to bending. So the theory which deals with stresses due to bending moment alone is termed as theory of simple bending. **5.2 Assumptions in the Theory of Simple Bending** 1. The material of the beam is homogeneous and isotopic. 2. The beam is initially straight and every layer of it is free to expand or contract. 3. The stress induced is proportional to strain and the stresses developed are within elastic limit. 4. Modulus of elasticity (*E*) is same for the material of beam in tension and compression. 5. The transverse sections of the beam which are plane, remain plane even after bending. 6. The radius of curvature of the beam is very large compared to its transverse dimensions.	3	7

X

4.4.6 Rankine's formula

We have seen that Euler's formula gives correct results only for very long columns. But what happens when the column is a short or the column is not a very long. On the basis of results of experiments performed bt Rankine, he established an emperical formula which is applicable to all columns whether they are short or long. The emperical formula given by Rankine is known as Rankines formula, which is given as 4

$$P = \frac{\sigma_c A}{1+\alpha\left(\frac{L}{k}\right)^2}$$

Where α = Rankines constant $= \frac{\sigma_c}{\pi^2 E}$

σ_c = Crushing stress of the material of the column 3

A = Cross-sectional area of the column

L = Equivalent length of the column 7

$\frac{L}{K}$ = Buckling factor $= \frac{\text{Equivalant length}}{\text{Least radius of gyration}}$

XI	Given: Outside diameter of hollow shaft, Do = 160 mm Inside diameter of hollow shaft, Di = 120 mm Maximum allowable shear stress, $\tau = 55$ MPa $= 55$ N/mm^2 Diameter ratio, $k = \frac{Di}{Do} = \frac{120}{160} = 0.75$ We know that strength of a shaft means the maximum torque that can be transmitted. Usin strength equation for hollow shaft. $\therefore$ Maximum torque transmitted, $T = \frac{\tau}{16}\pi D_o^3(1-k^4) = \frac{\pi}{16} \times 55 \times 160^3(1-0.75^4)$ $= 30237829.29$ N-mm $= 30.24$ kN-m (Ans)	5 1 1	
XII	Given Diameter of the cylinder, $d = 1\text{m} = 1 \times 10^3$ mm Internal pressure, $p = 2\text{N/mm}^2$. Hoop stress, $\sigma_h = 40\text{N/mm}^2$ Longitudinal stress $\sigma_l = 30\text{ N/mm}^2$. Considering hoop stress Thickness of the cylinder, $t = \frac{pd}{2\sigma_h} = \frac{2 \times 1000}{2 \times 40} = 25\text{mm}$ Considering Logitudinal stress $t = \frac{pd}{4\sigma_l} = \frac{2 \times 1000}{4 \times 30} = 16.67$ mm $\therefore$ $t = 25$ mm (Ans)		

<table>
<tr><td>XIII</td><td>Ass
5.
6.
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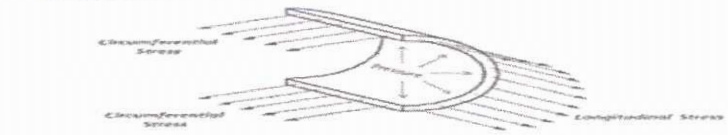
8.

G – Modulus of Rigidity
θ – Angle of twist
L – Length of shaft
τ – Shear stress
r – Radius of shaft

TORSION EQUATION

$$\frac{T}{I_p} = \frac{G\theta}{L} = \frac{\tau}{r}$$

T – Torque
I_p – Polar Moment of Inertia
G – Modulus of Rigidity
θ – Angle of twist
L – Length of shaft
τ – Shear stress
r – Radius of shaft</td><td>2
2
2</td><td></td></tr>
<tr><td>XIV</td><td>Stress in Cylindrical Component
Pressure in a cylinder always creates both;
- Circumferential stress (Hoop stress)
- Longitudinal stress



Longitudinal Stress
$$\sigma_2 = \frac{pr}{2t}$$


Hoop Stress
$$\sigma_1 = \frac{pr}{t}$$


Let η_l = Efficiency of logitudinal joint
η_h = Efficiency of circumfrential joint
Then circumfrential stress, $\sigma_h = \frac{pd}{2t\,\eta_l}$
and the Longitudinal stress, $\sigma_l = \frac{pd}{4t\,\eta_h}$

3.3.7 Design of thin cylindrical shells
It means to calculate the thickness of a cylindrical shell for the given length, diameters and intensity of maximum internal pressure.

Circumfrential stress, $\sigma_h = \frac{pd}{2t}$ or $\frac{pd}{2t\,\eta_l}$
$\therefore \quad t = \frac{pd}{2\sigma_h}$ or $\frac{pd}{2\sigma_h \eta_l}$
Longitudinal stress, $\sigma_l = \frac{pd}{4t}$ or $\frac{pd}{4t\,\eta_h}$
$\therefore \quad t = \frac{pd}{4\sigma_l}$ or $\frac{pd}{4\sigma_l \eta_h}$</td><td>1
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