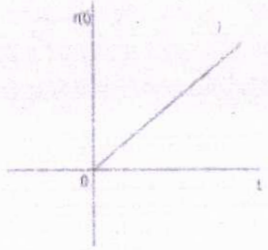
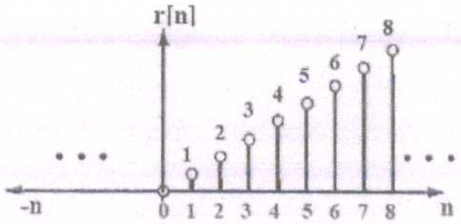
SCORING INDICATORS

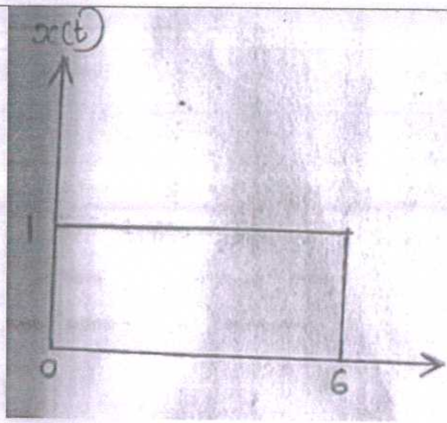
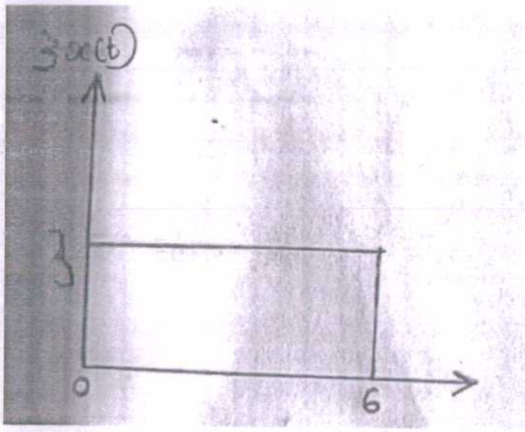
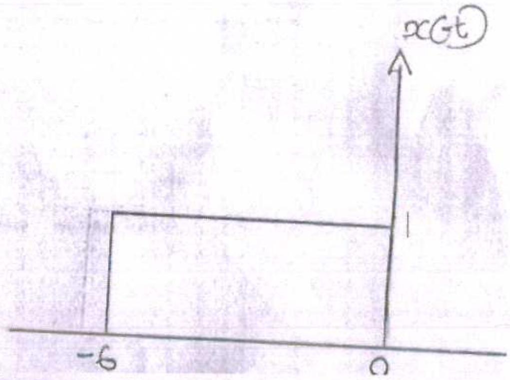
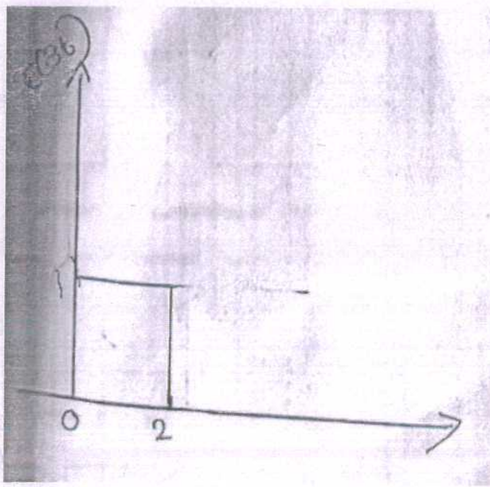
COURSE NAME:SIGNALS AND SYSTEMS
COURSE CODE: 5201 **QID:2109240233**

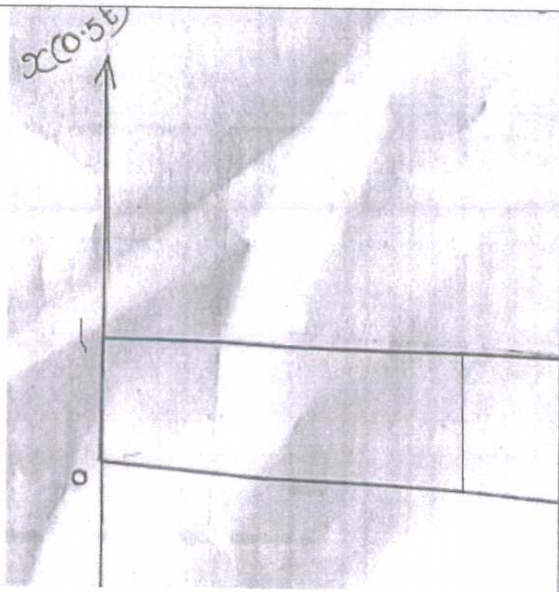
QNo	Scoring Indicator	Split score	Sub Total	Total score
	PARTA	1	1	9
I.1	$x(t) = A\sin(\omega t + \phi)$	1	1	
I.2	A random signal is a signal that varies unpredictably over time.	1	1	
I.3	An odd signal is a signal that satisfies the condition $x(-t) = -x(t)$. For discrete-time signals, it satisfies $x[-n] = -x[n]$.	1	1	
I.4	A differential equation representing a continuous-time system may take the form:$a_n \frac{d^n y(t)}{dt^n} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m x(t)}{dt^m} + \cdots + b_0 x(t)$	1	1	
I.5	A system is time-invariant if its behavior and characteristics do not change over time, i.e., the output is shifted in time if the input is shifted.	1	1	
I.6	The Nyquist rate is twice the maximum frequency component, so for a 10KHz signal, the Nyquist rate is $2 \times 10KHz = 20KHz$	1	1	
I.7	Fourier Series (FS) is used to evaluate the frequency domain representation of a continuous-time periodic signal.	1	1	
I.8	The Laplace Transform of the unit step function u(t) is $\frac{1}{s}$	1	1	
I.9	The inverse Laplace transform of $\frac{2}{s^2+4}$ is $\sin(2t)$	1	1	
	PART B			24
II.1	Continuous-time unit ramp signal:$r(t) = \begin{cases} t, t \geq 0 \\ 0, t < 0 \end{cases}$ 	1.5*2	3	

	Discrete-time unit ramp signal: $r[n]=\begin{cases}n, n \geq 0 \\ 0, n < 0\end{cases}$ 			
II.2	Real signal: A signal is real if its value is a real number for all time. Example: $x(t) = \cos(t)$ Imaginary signal: A signal is imaginary if its value is an imaginary number for all time. Example: $x(t) = j\sin(t)$	1.5*2	3	
II.3	Odd signal: $x(-t) = -x(t)$ Example: $x(t) = sin(t)$ Even signal: $x(-t) = x(t)$. Example: $x(t) = cos(t)$. Odd signals exhibit symmetry about the origin, while even signals exhibit symmetry about the vertical axis.	1.5*2	3	
II.4	Time shifting is the process of delaying or advancing a signal along the time axis. It alters the position of the signal without changing its shape. • Right shift (time delay): If x(t) is the original signal, a right shift (delaying the signal) by t_0 is represented as $x(t - t_0)$. This shifts the signal to the right on the time axis, delaying it by t_0 units. Effect: Each point on the signal occurs t_0 units later than it would in the original signal. • Left shift (time advance): A left shift (advancing the signal) by t_0 is represented as $x(t+t_0)$This shifts the signal to the left, advancing it by t_0 units.. Effect: Each point on the signal occurs t_0 units earlier than it would in the original signal.	Definition - 1 Explanation -2	3	

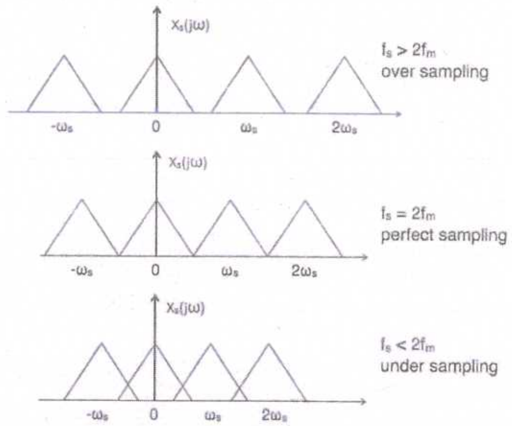
II.5	Continuous-Time Systems: 1. Definition: Process signals that are defined for all continuous values of time t. 2. Signal Representation: Represented as x(t), where t is a continuous variable. 3. Applications: Used in analog systems like electrical circuits and communication systems. Discrete-Time Systems: 1. Definition: Process signals defined only at discrete intervals of time nnn. 2. Signal Representation: Represented as x[n], where n is an integer representing time instants. 3. Applications: Used in digital systems like digital filters, signal processors, and computers.	1+1+1	3	
II.6	The impulse response characterizes a continuous-time linear time-invariant (LTI) system completely. It provides the output of the system when a unit impulse is applied at the input and can be used to compute the output for any arbitrary input using convolution.	3	3	
II.7	a) Time variant: $y(t) = x^2(t)$is time-variant because a time shift changes the form of the output. b) Time-invariant:$y(n) = x(n+1)$ is time-invariant since shifting the input results in a similar shift in the output.	1.5*2	3	
II.8	Fourier transform of unit impulse signal, $\delta(t) \to 1$ Fourier transform of unit step signal $u(t) \to \frac{1}{j\omega} + \pi\delta(\omega)$ Fourier transform of cosine signal $\cos(\omega_0 t) \to \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	1*3	3	
II.9	a) Right-sided exponential signal: ROC is Re(s) > α. b) Left-sided exponential signal: ROC is Re(s) < α. c) Two-sided exponential signal: ROC is α1< Re(s) <α2		3	
II:10	The linearity property of the Laplace Transform states that if $x_1(t) \to X_1(s)$ and $x_2(t) \to X_2(s)$, then for any constants a and b,$ax_1(t) + bx_2(t) \to aX_1(s) + bX_2(s)$	1*3	3	
	PARTC			42
III	• Energy Signals:	Energy signals-3.5	7	

	• Definition: A signal is classified as an energy signal if its total energy is finite over all time. These signals typically decay to zero as $t \to \infty$ • Example: The exponentially decaying signal $x(t) = e^{-t}u(t)$) has finite energy, making it an energy signal. The integral of $\mid x(t) \mid^2$ over time converges to a finite value. • Power Signals: • Definition: A signal is classified as a power signal if it has infinite energy but finite average power. These signals usually oscillate or repeat over time without decaying. • Example: The sinusoidal signal $x(t) = \cos(\omega t)$ oscillates indefinitely but has finite average power. • Example Calculations: • For energy signals, the total energy is calculated by integrating the square of the signal over all time. For the signal $x(t) = e^{-t}u(t)$, the energy converges to $\frac{1}{2}$. $E = \int_{-\infty}^{\infty} \mid x(t) \mid^2 \, dt$ $E = \int_{0}^{\infty} e^{-2t} dt = \frac{1}{2}$ • For power signals, the average power is calculated as the mean square value over one period. For the signal $x(t) = \cos(\omega t)$, the power is $\frac{1}{2}$, constant over time. $P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \mid x(t) \mid^2 \, dt$ $P = \frac{1}{2\pi} \int_{0}^{2\pi} \cos^2(\omega t) dt = \frac{1}{2}$	Power signals-3.5 Definition-2 Explanation -2 Calculation-3		

IV		 a)  b)  c)  d)	1+1+1+2+2	7	

	e) 			
V	a) $y(t) = x(t) + 3$ • Linearity Test: A system is linear if it satisfies both additivity and homogeneity. • Additivity: If the input is $x_1(t) + x_2(t)$, the system output becomes: $y(t) = (x_1(t) + x_2(t)) + 3 = x_1(t) + x_2(t) + 3$ This is not equal $y_1(t) + y_2(t)$, so the system fails additivity. • Homogeneity: For a scalar a, if the input is $a \cdot x(t)$, the system output becomes: $y(t) = a \cdot x(t) + 3$ This is not equal $a \cdot y(t) = a \cdot (x(t) + 3)$, so the system fails homogeneity. • Conclusion: The system is non-linear. b) $y(t) = \frac{d}{dt}x(t)$ • Additivity: The derivative of a sum of inputs is: $y(t) = \frac{d}{dt}(x_1(t) + x_2(t)) = \frac{d}{dt}x_1(t) + \frac{d}{dt}x_2(t) = y_1(t) + y_2(t)$Thus, additivity holds. • Homogeneity: For a scalar a, the derivative of a scaled input is: $y(t) = \frac{d}{dt}(a \cdot x(t)) = a \cdot \frac{d}{dt}x(t) = a \cdot y(t)$ Thus, homogeneity holds. • Conclusion: The system is linear. c) $y(t) = e^{x(t)}$ • Additivity: For $y(t) = e^{x_1(t)+x_2(t)}$, we see: $y(t) = e^{x_1(t)+x_2(t)} \neq e^{x_1(t)} + e^{x_2(t)}$	1+2+2+2	7	

	Thus, the system fails additivity. • Homogeneity: For a scalar a, the output becomes: $y(t) = e^{a \cdot x(t)} \neq a \cdot e^{x(t)}$ Thus, the system fails homogeneity. • Conclusion: The system is non-linear. d) $y(t) = \cos(x(t))$ • Additivity: $y(t) = \cos(x_1(t) + x_2(t))$, we see: $y(t) = \cos(x_1(t) + x_2(t)) \neq \cos(x_1(t)) + \cos(x_2(t))$ Thus, the system fails additivity. • Homogeneity: For a scalar a, the output becomes: $y(t) = \cos(a \cdot x(t)) \neq a \cdot \cos(x(t))$.Thus, the system fails homogeneity. Conclusion: The system is non-linear.			
VI	• Linearity: • A system is linear if it satisfies additivity (output for $x_1(t) + x_2(t)$is the sum of individual outputs) and homogeneity (output for $a \cdot x(t)$ is $a \cdot y(t)$. • Example: $y(t) = 3x(t)$ is linear, but $y(t) = x^2(t)$ is non-linear. • Time-Invariance: • A system is time-invariant if a time shift in the input results in an identical time shift in the output. • Example: $y(t) = 2x(t)$ is time-invariant, while $y(t) = x(2t)$ is time-variant. • Causality: • A system is causal if the output depends only on present and past inputs, not future values. • Example: $y(t) = x(t) + x(t-1)$ is causal, but $y(t) = x(t) + x(t+1)$ is non-causal. • Stability: • A system is stable if a bounded input results in a bounded output (BIBO Stability). • Example: $y(t) = 0.5x(t)$ is stable, while $y(t) =$	Linearity-1 Time invariance-2 Causality-2 Stability-2 (Equation-1,Explanation-1)	7	

	$2^t x(t)$ is unstable.			
VII	• Sampling Theorem: • The Sampling Theorem states that a signal can be exactly reconstructed from its samples if it is sampled at a rate greater than twice the maximum frequency of the signal, known as the Nyquist rate. • Nyquist Rate: $f_s \geq 2f_{\max}$, where f_s is the sampling rate and $f_{\max}$ is the highest frequency in the signal. • Aliasing: • Aliasing occurs when the sampling rate is below the Nyquist rate, causing different signals to become indistinguishable or "fold" into each other. • This results in high-frequency components appearing as lower frequencies in the sampled signal.  • Impact of Incorrect Sampling: • If the signal is under sampled, it leads to loss of information and distortion in the reconstructed signal, making it impossible to recover the original signal accurately. • Example: A 10 kHz signal sampled below 20 kHz will exhibit aliasing, where the high-frequency components will appear incorrectly in the sampled data.	Sampling Theorem-2 Aliasing-4(fig-2+explanation-2) Impact- 2	7	
VIII	a) Linearity: • The DTFT is linear, meaning if $x_1[n]$ and $x_2[n]$ are two signals and a and b are constants, then:		7	

	$\text{DTFT}\{ax_1[n] + bx_2[n]\} = aX_1(e^{j\omega}) + bX_2(e^{j\omega})$ • Illustration: If $x_1[n] = \delta[n]$ and $x_2[n] = u[n]$, their combined transform is a sum of individual transforms. b) Time Shifting: • If a signal $x[n]$ is shifted by n_0, the DTFT of the shifted signal is: $$\text{DTFT}x[n - n_0] = X(e^{j\omega})e^{-j\omega n_0}$$ Illustration: A time shift introduces a phase shift in the frequency domain. c) Frequency Shifting: • Multiplying a signal by a complex exponential in the time domain shifts the frequency components: $\text{DTFT}\{x[n]e^{j\omega_0 n}\} = X(e^{j(\omega-\omega_0)})$ • Illustration: Shifting the frequency of a signal modifies its spectrum. d) Convolution Property: • The convolution of two signals in the time domain corresponds to the multiplication of their DTFTs: $\text{DTFT}x_1[n] * x_2[n] = X_1(e^{j\omega}) \cdot X_2(e^{j\omega})$ • Illustration: Useful for filtering, where convolving a signal with a filter's impulse response corresponds to multiplying in the frequency domain			
IX	a) Linearity: • The Fourier series is linear, meaning if two signals and $x_2(t)$) are combined with constants a and b, then: $ax_1(t) + bx_2(t) \Rightarrow aC_1 + bC_2$ Example: Superposition of signals results in the superposition of their Fourier coefficients. b) Time Shift: • Shifting a signal in time by t_0 introduces a phase shift in the Fourier series coefficients: $$x(t - t_0) \Rightarrow C_n e^{-jn\omega_0 t_0}$$ Example: A time shift causes a corresponding phase	1+2+2+2 (equation-1 +explanation-1 for each)	7	

	shift in the frequency domain. c) Frequency Shift: • Multiplying a signal by a complex exponential in the time domain results in a frequency shift in the Fourier series $x(t)e^{j\omega_0 t} \Rightarrow C_{n-k}$ Example: Shifting a signal's frequency changes the position of its Fourier coefficients. d) Multiplication in Time Domain: • Multiplying two signals in the time domain results in the convolution of their Fourier coefficients: $x_1(t) \cdot x_2(t) \Rightarrow C_1 * C_2$ • Example: This property is useful in modulation, where time-domain multiplication spreads the frequency content.			
X	a) Fourier Transform of the Dirac Delta Function δ(t): • The Fourier Transform is given by: • $\mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt = 1$ • Explanation: The delta function δ(t) only exists at t=0, so the integral evaluates to 1. This means the Dirac delta function contains all frequencies equally, resulting in a flat spectrum in the frequency domain. b) Fourier Transform of $e^{-a\|t\|}$ (Exponential Decay Function): • The Fourier Transform is: • $\mathcal{F}\{e^{-a\|t\|}\} = \int_{-\infty}^{\infty} e^{-a\|t\|}e^{-j\omega t}dt = \frac{2a}{a^2+\omega^2}$ • Explanation: This is an even, symmetric function, decaying on both sides of t=0. The Fourier transform results in a rational function, indicating that the frequency response of the signal is low-pass, meaning high-frequency components are attenuated. c) Fourier Transform of $\sin(\omega_0 t)$ (Sine Function): • The Fourier Transform is: • $\mathcal{F}\sin(\omega_0 t) = \int_{-\infty}^{\infty} \sin(\omega_0 t)\, e^{-j\omega t}dt =$	Dirac delta function - 1(calculation) Exponential decay function-3(eqn-1+calculation-2) Sine function-3 (eqn-1+calculation-2)	7	

	$\frac{j}{2}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$ • Explanation: The sine function results in two delta functions in the frequency domain, located at $\pm\omega_0$ This means the signal has energy concentrated at these two frequencies, corresponding to its oscillatory nature.		
XI	• Unit Step Function u(t): • $\mathcal{L}(u(t))=\frac{1}{s}$, for $\Re(s)>0$ • Exponential Function e^{at}: $\mathcal{L}\{e^{at}\}=\frac{1}{s-a}$, for $\Re(s)>a$ • Sine Function $\sin(\omega t)$ • $\mathcal{L}\{\sin(\omega t)\}=\frac{\omega}{s^2+\omega^2}$, for $\Re(s)>0$ • Cosine Function $\cos(\omega t)$ $\mathcal{L}\{\cos(\omega t)\}=\frac{s}{s^2+\omega^2}$, for $\Re(s)>0$ • Ramp Function t: $\mathcal{L}\{t\}=\frac{1}{s^2}$, for $\Re(s)>0$ • Dirac Delta Function $\delta(t)$ $\mathcal{L}\{\delta(t)\}=1$ • Exponential Decay $e^{-at}u(t)$ $\mathcal{L}\{e^{-at}u(t)\}=\frac{1}{s+a}$, for $\Re(s)>-a$	1*7 (1 mark for each signal)	7
XII	a) Inverse Laplace Transform of $\frac{1}{(s+1)(s+2)}$: ○ Use partial fraction decomposition: $\frac{1}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$ ○ Solving for A and B, we get: $\frac{1}{(s+1)(s+2)}=\frac{1}{s+1}-\frac{1}{s+2}$ Inverse Laplace Transform:	3+1+3	7

	$$\mathcal{L}^{-1}\{\frac{1}{s+1}-\frac{1}{s+2}\}=e^{-t}-e^{-2t}$$ b) Inverse Laplace Transform of $\frac{2s}{s^2+1}$: • Recognize this as the Laplace transform of a cosine function: • $\mathcal{L}^{-1}\{\frac{2s}{s^2+1}\}=2\cos(t)$ c) $X(s)=\frac{s+3}{s^2+6s+13}$ $$s^2+6s+13=(s+3)^2+4$$ $$\frac{s+3}{(s+3)^2+4}=\frac{s+3}{(s+3)^2+2^2}$$ By applying frequency shifting property, $$\mathcal{L}^{-1}\{\frac{s+3}{(s+3)^2+4}\}=e^{-3t}\cos(2t)$$			
XIII	• The ROC of the Laplace transform is the range of values of s (in the complex plane) for which the Laplace transform integral converges. • It defines where the Laplace transform is valid. • ROC for Stable Systems: • A system is stable if the ROC includes the imaginary axis $\Re(s)=0$. • Stability means that the system produces bounded output for any bounded input. • Example: For $e^{-at}u(t)$, the ROC is $\Re(s)>-a$, and for the system to be stable, the imaginary axis must be within the ROC. • ROC for Causal Systems: • A system is causal if the ROC is to the right of the rightmost pole.	Definition of ROC-2 ROC for Stable Systems-3(explanation-2+example-1) ROC for Causal Systems-3(explanation-2+example-1)	7	

	• Causal systems depend only on present and past inputs (not future). • Example: For $e^{-at}u(t)$, the ROC is $\Re(s) > -a$, ensuring causality.			
XIV	a) Time Reversal: • Property: If $x(t)$ has a Laplace transform $X(s)$, then the Laplace transform of $x(-t)$ is $X(-s)$. • Explanation: Reversing the signal in time corresponds to negating the s-variable in the Laplace domain. b)Time Shifting: • Property: If $x(t)$ has a Laplace transform $X(s)$, then the Laplace transform of $x(t-t_0)$ is: • $\mathcal{L}\{x(t-t_0)\} = e^{-st_0}X(s)$ • Explanation: Shifting a signal in time by t_0 introduces a multiplicative exponential factor in the Laplace domain. c) Convolution in Time Domain: • Property: The convolution of two signals $x_1(t)$and $x_2(t)$ in the time domain corresponds to the multiplication of their Laplace transforms in the frequency domain: • $\mathcal{L}\{x_1(t) * x_2(t)\} = X_1(s)X_2(s)$ • Explanation: Convolution in time becomes a simple multiplication in the Laplace domain, simplifying the analysis of systems. d) Final Value Theorem: • Property: The final value of a signal $x(t)$ as $t \to \infty$ can be determined using its Laplace transform: $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$ • Explanation: This theorem helps find the steady-state value of a system without needing the inverse Laplace transform.	1+2+2+2 (equation-1+explanation-1 for each)	7	