Answer Key

50P3 ①

Part A

1) A system in which the controlling action or input is dependent on the output or changes in output is called closed loop control system.

2) Transfer function is expressed as the ratio of o/p quantity to i/p quantity.
if expressed in L.T it is the ratio of L.T of o/p variable to L.T of i/p variable assuming all initial conditions are zero.

3) Breakaway point is the point on the root locus where multiple roots of the characteristic equation occur for a particular value of K.

4) Type of the system means no. of poles at origin of open loop transfer function $G(s)H(s)$ of the system.

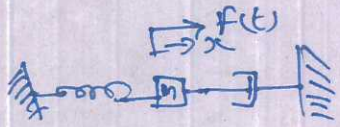
5) The phase margin of a stable system is the amount of additional phase lag required to bring the system to the point of instability.

Part B

II) 1)

$$L.T \text{ of } e^{at} = \int_0^\infty e^{at} \cdot e^{-st}\, dt = \frac{1}{(s-a)}.$$

$$L.T \text{ of } \sin at = \frac{a}{s^2+a^2}.$$

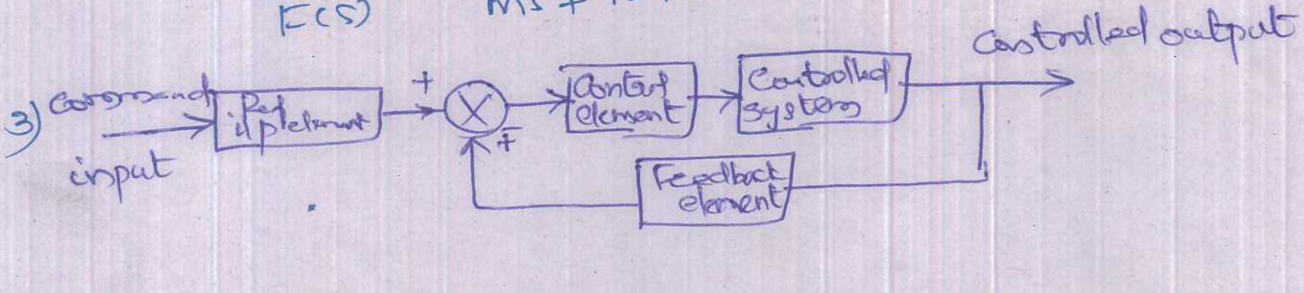


2) Inertia force = Force acting on mass.

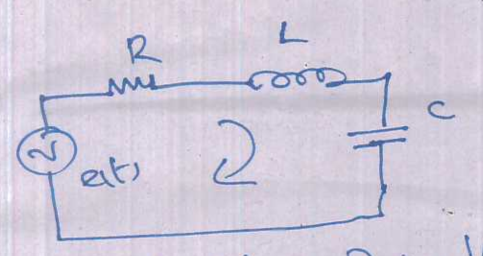
$$M\frac{d^2x}{dt^2} = -f\frac{dx}{dt} - Kx + f(t)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{1}{Ms^2+fs+K}.$$

3)



4)



$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\, dt = e(t)$$

$$i = \frac{dq}{dt}$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = e(t).$$

1) Applied force $f(t)$ is analogous to applied voltage $e(t)$
2) Mass M is analogus to L.
3) Coefficent of various friction f is analogue to R
4) Spring deflection constant k is analogus to $1/c$.
5) Displacement x is analogus to q.

$L\frac{d^2q}{dt^2}$, $R\frac{dq}{dt}$, $\frac{1}{c}q$, $e(t)$ are voltages and $M\frac{d^2x}{dt^2}$, $f\frac{dx}{dt}$, Kx, $f(t)$ forces therefore, aforesaid analogy is called force voltage analogy.

5) Static positional error coefficient

(i) $e_{ss} = \underset{s\to 0}{Lt}\, sR(s)\frac{1}{1+G(s)H(s)} = \frac{1}{1+K_p}$.

$R(s) = 1/s$; $K_p = \underset{s\to 0}{Lt}\, G(s)H(s)$.

(ii) Static velocity error coefficeints

$e_{ss} = \underset{s\to 0}{Lt}\, s\, R(s)\frac{1}{1+G(s)H(s)}$

$R = 1/s^2$

$e_{ss} = 1/K_v$ where $K_v = \underset{s\to 0}{Lt}\, s\,G(s)H(s)$

(iii) Static acceleration error coefficient

$e_{ss} = \underset{s\to 0}{Lt}\, s\, R(s)\frac{1}{1+G(s)H(s)}$

$R = 1/s^3$

$e_{ss} = \underset{s\to 0}{Lt}\frac{1}{s^2G(s)H(s)}$

$K_a = \underset{s\to 0}{Lt}\, s^2G(s)H(s)$

$\therefore e_{ss} = 1/K_a$.

b) Case 1

~~Any~~ One element in the first column becomes ~~zero~~ zero. This is solved by replacing zero by a symbol 'ε' and Routh array is formed.

Case 2

All the elements of a row in Routh's table becoming zero. Take the elements above the row having all zeroes and write auxilary equation. differentiate this equation and place the coefficient obtained ~~for~~ of $\frac{dA(s)}{ds}$ ~~is the~~ for zero. Then solve.

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(vii) Gain Cross over frequency

The frequency at which magnitude of $G(j\omega)H(j\omega)$ is unity

Phase Cross over frequency ~~Fpc~~

The frequency at which phase angle of $G(j\omega)H(j\omega)$ is -180^0.

(i) If phase cross over frequency is greater than gain cross over frequency, then control system is stable.

(ii) If PCF is equal to ~~PCF~~ GCF then control system is marginally stable.

(iii) If PCF < GCF, then control system is unstable.

Part C

III)(a) Open loop

Accuracy depends on calibration of input. Any variation from pre-determind calibration affects the o/p.

Open-loop system is simple to construct and cheap.

Open-loop systems are generally stable.

Open-loop system affected due to the operation of open-loop system affected due to the presence of non-linearities in its elements.

Closed loop

Closed loop systems works more accurately.

Closed loop system is complicated to construct and costly.

Closed loop system can become unstable under certain conditions.

Closed loop system adjusts to the effects of non linearities present in its elements.

b) If L.T of $f(t)$ is $F(s)$ then

$$L\frac{df(t)}{dt} = [sF(s) - f(0+)]$$

$$L\frac{d^2f(t)}{dt^2} = [s^2F(s) - sf(0+) - f'(0+)]$$

$$L\frac{d^3f(t)}{dt^3} = [s^3F(s) - s^2f(0+) + sf'(0+) - f''(0+)]$$

where $f(0+), f'(0+), f''(0+)$ are the values of $f(t), \frac{df(t)}{dt}, \frac{d^2f(t)}{dt^2}$ - - - - at $t = (0+)$.

[Statement = 3
Proof = 4]

2

IV) a) ~~L.T of $R_1 = R_1$~~

~~$L = sL$~~

~~$C = 1/sC$~~

~~$e_i(t) \to E_i(s)$ $e_o(t) \to E_o(s)$~~

~~$$\frac{E_o(s)}{E_i(s)} = \frac{1}{sC\left[R + sL + \frac{1}{sC}\right]} = \frac{1}{RsC + s^2LC + 1}$$~~

L.T of integral of f(t)

$$\mathcal{L}\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$$

where F(s) is L.T of f(t) and $f^{-1}(0) = \int f(t)\,dt$ at $t=0$.

(statement 3 + Proof 4)

b) $F(s) = \frac{1}{s^2+4s+8}$

$s^2+4s+8 = (s+2)^2 + 2^2$

$$\mathcal{L}^{-1} F(s) = \mathcal{L}^{-1} \frac{1}{[(s+2)^2+(2^2)]}$$

$$= \mathcal{L}^{-1} \frac{1}{2} \cdot \frac{2}{(s+2)^2+(2^2)}$$

$$\therefore f(t) = \frac{1}{2} e^{-2t} \sin 2t$$

V) b)

$L_1 = G_4(-H_1)$ $L_2 = G_2G_4(-H_2)$ $L_3 = G_2G_5(-H_2)$.

$\Delta = 1 + G_4H_1 + G_2G_4H_2 + G_2G_5H_2$

$P_1 = G_1G_2G_4 \cdot 1$ $P_2 = G_1G_2G_5 \cdot 1$ $P_3 = G_3$.

$L_1 = -G_4H_1$ not touching path P_3.

$\therefore \Delta_3 = (1+L_1) = [1 + (-G_4H_1)] = 1 + G_4H_1$

$$\frac{C}{R} = \frac{G_1G_2G_4 + G_1G_2G_5 + G_3(1+G_4H_1)}{1 + G_4H_1 + G_2G_4H_2 + G_2G_5H_2}$$

VI) a) ① shift summing point before block G_1 to a position after block G_1

② Interchange consecutive summing points after G_1.

③ eliminate summing point after block G_1.

④ shift the summing point after block (G_1+G_2) to a position before block (G_1+G_2)

⑤ combine blocks (G_1+G_2) and G_3.

$$\therefore \text{Transferfunction} = \frac{C}{R} = \frac{(G_1+G_2)G_3}{1 + (G_1+G_2)G_3 \cdot \frac{G_1H_1}{(G_1+G_2)}}$$

ie, $\frac{C}{R} = \frac{G_1G_3 + G_2G_3}{1 + G_1G_3H_1}$

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②

b) Source Node
Sink node
Chain node
Forward Path
Feedback loop
Self loop
Path gain
Non touching loops
Loop gain
Dummy node - explanation for any 5 of these terminologies
(5×1=5).

vii) $G(s)H(s) = \dfrac{K(1+sT_a)(1+sT_b)\ldots}{(1+sT_1)(1+sT_2)\ldots}$

$K_p = \lim_{s\to 0} G(s)H(s)$

$= \lim_{s\to 0} \dfrac{K(1+sT_a)(1+sT_b)\ldots}{(1+sT_1)(1+sT_2)\ldots}$

$\therefore K_p = K.$

$e_{ss} = 1/(1+K_p)$

$= 1/(1+K)$

$K_v = \lim_{s\to 0} s\,G(s)H(s)$

$= \lim_{s\to 0} s\, \dfrac{K(1+sT_a)(1+sT_b)\ldots}{s(1+sT_1)(1+sT_2)\ldots}$

$K_v = 0$

$\therefore e_{ss} = 1/K_v$

$= 1/0 = \infty.$

$K_a = \lim_{s\to 0} s^2\, G(s)H(s)$

$= \lim_{s\to 0} s^2\, \dfrac{K(1+sT_a)(1+sT_b)\ldots}{s(1+sT_1)(1+sT_2)\ldots}$

$K_a = 0$

$e_{ss} = 1/K_a = 1/0 = \infty$

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VI(1)(a) Absolute Stability

A system whose output does not change with variations in parameters, disturbances etc is called absolute stable system and such stability is known as absolute stability.

Relative stability

stability depends on certain conditions of the system.

Margin stability

Systems which keep on oscillating when certain force is applied. Such systems are neither stable nor unstable and hence called marginally stable system. eg: pendulum.

b) $s^6 + s^5 + 5s^4 + 3s^3 + 2s^2 - 4s - 8 = 0$.

s^6	1	5	2	-8
s^5	1	3	-4	0
s^4	2	6	-8	0
s^3	0	0	0	0
s^2				

$\Rightarrow$ Aux equation is

$A(s) = (2s^4 + 6s^2 - 8)$

$\frac{dA(s)}{dt} = (8s^3 + 12s - 0)$

s^6	1	5	2	-8
s^5	1	3	-4	0
s^4	2	6	-8	0
s^3	8	12	0	0
s^2	3	-8	0	0
s^1	$\frac{100}{3}$	0	0	0
s^0	-8	0	0	0

There is one sign change in the first column.

$\therefore$ The system is unstable.

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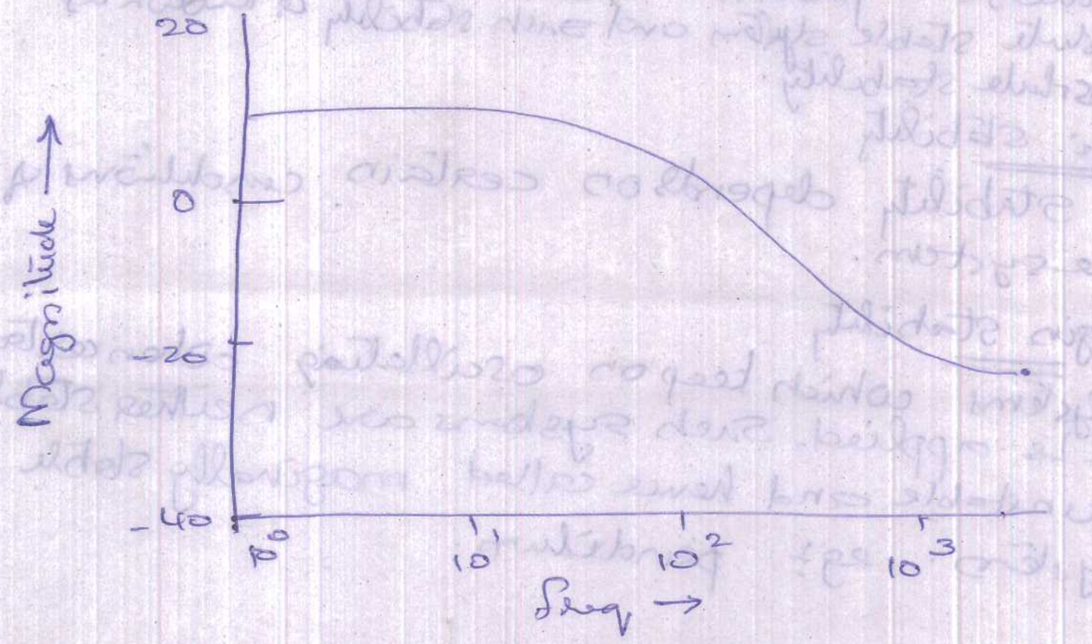
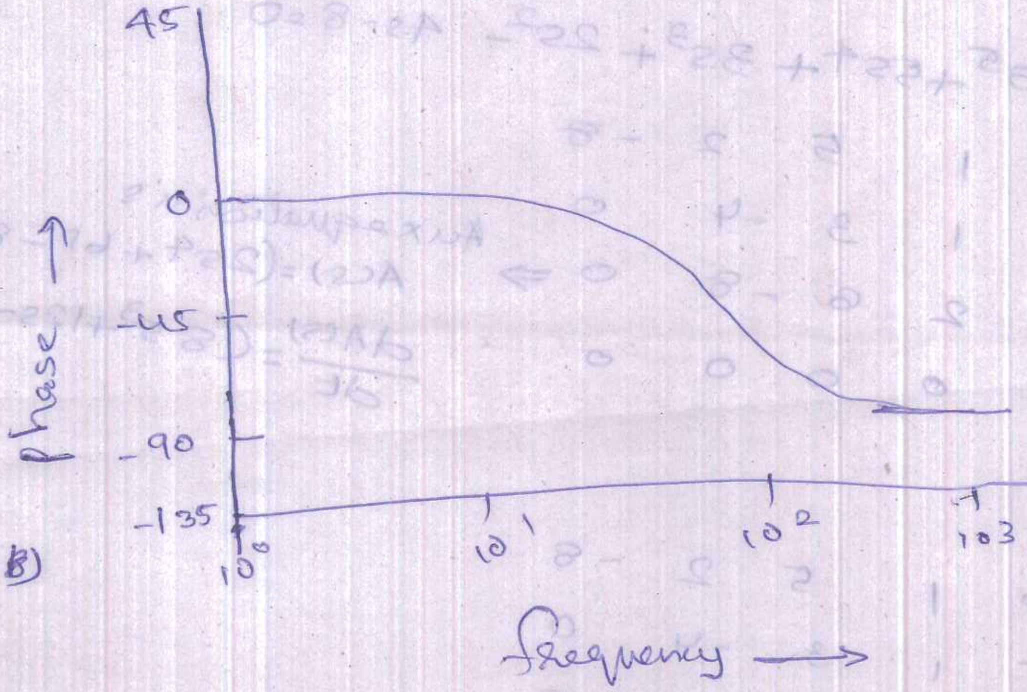
(A)

IX) a)





B)

Xb) Gain Margin

is defined as the margin in ~~gan~~ gain allowable by which gain can be increased till system reaches on the verge of instability. Gain margin is expressed as the ~~reciprocal~~ reciprocal of the magnitude of $G(j\omega)H(j\omega)$ at phase cross over frequency

$$G.M = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{cp}}}$$

where ω_{cp} = phase cross over frequency

Generally GM is expressed in decibels.

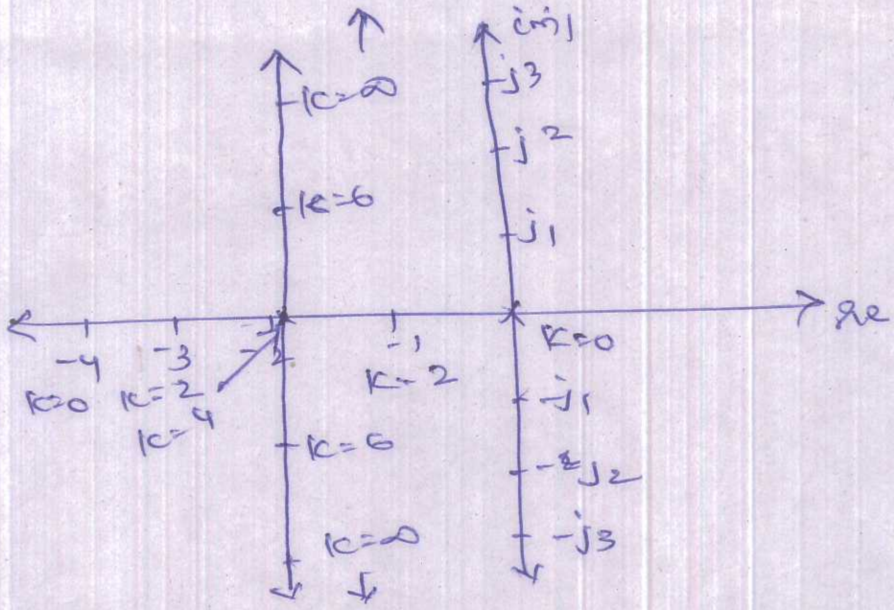
X) a) $H(s) = 1.$

$1 + G(s)H(s) = 0$

$s^2 + 4s + K = 0$

roots are $s_1 = s_2 = -2 \pm \sqrt{4-K}$

K	0	2	4	6	∞
s_1	0	-1	-2	$-2+j\sqrt{2}$	$-2+j\infty$
s_2	-4	3	-2	$-2-j\sqrt{2}$	$-2-j\infty$



No. of open loop poles P = 2

No. of open loop zeros Z = 0

No. of root locus branches = N = 2

The system is stable.

IX) b) (a) ~~1st~~ starting points

(b) ending points

(c) No of branches

(d) Existence on real axis

(e) Break away points

(f) angle of asymptots $\frac{(2k+1)180^\circ}{P-Z}$ $k = 0, 1, 2, \ldots (P-Z)-1$

(g) Intersection of asymptos on real axis

(h) Intersection points in imaginary axis $\quad x = \frac{\sum \text{poles} - \sum \text{zeros}}{P-Z}$

(i) Angle of departure from complex ~~path~~ pole.

$\theta_d = 180^\circ - (\phi_p - \phi_z)$

(j) Angle of arrival at complex zeros

$\phi_a = 180 - (\phi_z - \phi_p).$

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