SCHEME OF VALUATION

Revision: 2015

Course code: **5043**

Course Title: **CONTROL SYSTEMS**

PART A

1. Differential equation model, transfer function model and state space model. (2)
2. The order of the system is the order of the differential equation governing the system. It is also the maximum power of s in the denominator polynomial of system transfer function. (2)
3. Steady state error is the value of error signal e(t) when t tends to infinity. (2)
4. The frequency response is a steady state output of the system when the input is a sinusoidal signal. (2)
5. The path taken by a root of characteristic equation when open loop gain K is varied from 0 to infinity is called root locus. (2)

PART B

II 1. Systems that are both linear and time invariant are said to be linear time invariant systems. In linear systems the output varies linearly with input. They satisfy both homogeneity and superposition. If the output of the system is independent of the time at which the input is applied it is called time invariant systems.

If the output of the system varies depends on the time at which input is applied it is called time variant systems, systems that are linear but time variant are called linear time variant systems. (6)

2. $f(t) = \sin \omega t$

$F(s) = L \sin(\omega t) = \int_0^\infty \sin\omega t \, e^{-st} dt$

Substitute $\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$ (2)

$\therefore F(s) = \int_0^\infty \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \times e^{-st} dt$

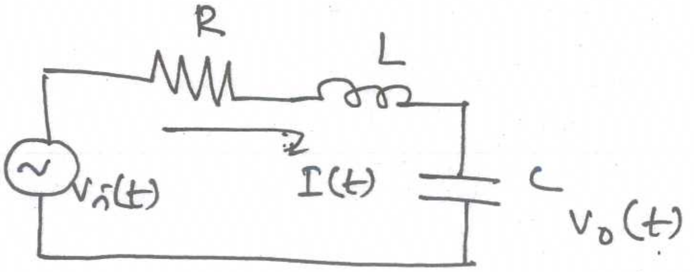
$= \int_0^\infty e^{-(s-j\omega)t} - e^{-(s+j\omega)t} dt$ (2)

$$F(s) = \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

$$= \frac{1}{2j}\left[\frac{s+j\omega-(s-j\omega)}{s^2-(j\omega)^2}\right] \qquad (6)$$

$$= \frac{1}{2j}\,\frac{2j\omega}{s^2+\omega^2} = \underline{\underline{\frac{\omega}{s^2+\omega^2}}} \qquad (2)$$

3.



1. model equations.

$$V_i(t) = R\,I(t) + L\frac{dI}{dt} + \frac{1}{C}\int I\,dt$$

$$V_o(t) = \frac{1}{C}\int I\,dt \qquad (2)$$

2. Input & output variables

input $V_i(t)$ output $V_o(t)$

3. Laplace Transform (assuming zero initial conditions)

$$V_i(s) = R\,I(s) + sL\,I(s) + \frac{1}{sC}I(s)$$

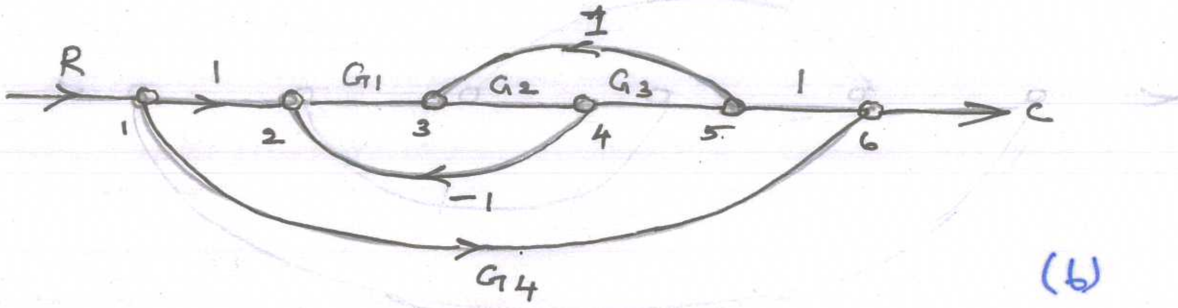
$$V_o(s) = \frac{1}{sC}I(s) \qquad (2)$$

4. Transfer function

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}I(s)}{R\,I(s) + sL\,I(s) + \frac{1}{sC}I(s)}$$

$$G(s) = \frac{1}{s^2LC + sRC + 1}$$ (2) (6)

4.



(6)

5. The K_p, K_v and K_a are called static error constants.

The positional error constant $K_p = \lim_{s \to 0} G(s)H(s)$. The steady state error in type 0 system when the input is unit step is given by $\frac{1}{1+K_p}$ (2)

The velocity error constant $K_v = \lim_{s \to 0} s\, G(s)H(s)$. The steady state error in type 1 system when the input is unit ramp input is given by $\frac{1}{K_v}$ (6)

(2)

The acceleration error constant $K_a = \lim_{s \to 0} s^2 G(s)H(s)$ The steady state error in type 2 system when the input is unit parabolic input is given by $\frac{1}{K_a}$ (2)

6. Absolute stability means whether system is stable or unstsble. (2)

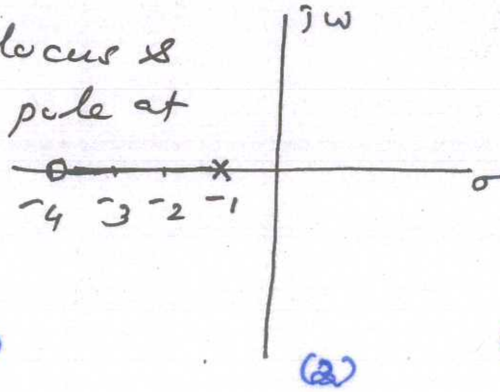
Relative stability gives the degree of stability or how close it is to instability. (2) (6)

In control theory a linear time invariant system is marginally stable if it is neither asymptotically stable nor unstable. (2)

7. $G(s) = \frac{s+4}{s+1}$ open loop pole m is at $s = -1$ (1)

open loop zero m is at $s = -4$ (1)

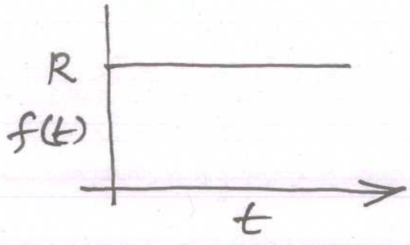
There is only one root locus & it start from open loop pole at $s = -1$, terminates at open loop zero at $s = -4$. (2)



(2) (6)

Part C

III 1. Step input



For a step input

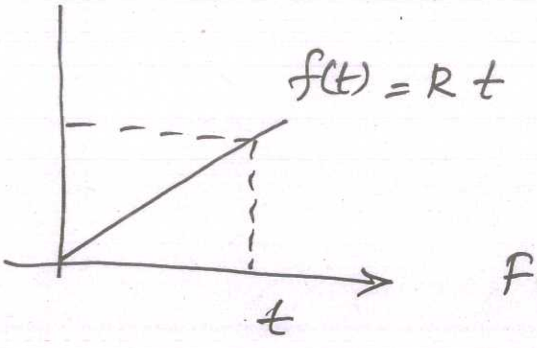
$f(t) = 0$ for $t < 0$

$f(t) = R$ for $t > 0$

$$F(s) = \mathcal{L} f(t) = \int_0^\infty R \,.\, e^{-st}\, dt$$

$$= R \,.\, \frac{e^{-st}}{-s}\Big|_0^\infty = \underline{\underline{\frac{R}{s}}} \qquad (4)$$

Ramp input [velocity function]



$f(t) = 0$ for $t < 0$

$f(t) = Rt$ for $t > 0$

$$F(s) = \int_0^\infty R t\, e^{-st}\, dt$$

$$F(s) = R t \frac{e^{-st}}{-s}\Big|_0^\infty - \int_0^\infty R \,.\, 1 \,.\, \frac{e^{-st}}{-s}\, dt \qquad (10)$$

$$= \frac{R}{s}\int_0^\infty e^{-st}\, dt = \frac{R}{s}\, \frac{e^{-st}}{-s}\Big|_0^\infty$$

$$= \frac{-R}{s^2}\,[0-1] = \underline{\underline{\frac{R}{s^2}}} \qquad (6)$$

III 2.

Sl.No.	Open loop system	Closed loop system
1	These are not reliable	These are reliable
2	It is easier to build	It is difficult to build
3	If calibration is good, they perform accurately	They are accurate because of feedback
4	Open loop systems are generally more stable	These are less stable
5	Optimization is not possible	Optimization is possible

IV 1. $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 12$ (5)

$s^2Y(s) - s\,y(0) - \dot{y}(0) + 5s\,y(s) - y(0)$

$+ 6y(s) = \frac{12}{s}$ (2)

$s^2Y(s) + 5sY(s) + 6Y(s) = \frac{12}{s}$

$Y(s) = \frac{12}{s(s^2+5s+6)} = \frac{12}{s(s+2)(s+3)}$ (2)

$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$

$A = s \times Y(s)\Big|_{s=0} = 2$

$B = (s+2)\,Y(s)\Big|_{s=-2} = -6$

$C = (s+3)\,Y(s)\Big|_{s=-3} = 4$ (4)

$\therefore Y(s) = \frac{2}{s} + \frac{-6}{s+2} + \frac{4}{s+3}$ (10)

$Y(s) = 2 - 6e^{-2t} + 4e^{-3t}$ (2)

IV a Differentiation theorems

The L.T of derivative of a function f(t)

$L\frac{d}{dt}f(t) = s\,f(s) - f(0)$, f(0) is initial value of f(t)

$L\frac{d^2}{dt^2}f(t) = s^2 f(s) - s\,f(0) - f'(0)$. (2.5)

Integration theorems

(5)

$L\int f(t)\,dt = \frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$.

$L\iint f(t)\,dt = \frac{F(s)}{s^2} + \frac{F^{-1}(0)}{s^2} + \frac{f^{-1}(0)}{s}$ (2.5)

V 1. **Node**: a node is a point representing a variable or signal.

Branch: a branch is a directed line segment joining two nodes.

Transmittance : the gain acquired by the signal when it travels from one node to another is called transmittance.

Input node (source): it is anode that has only outgoing branches.

Output node (sink): it is a node that has only incoming branches.

Mixed node: it is a node that has both incoming and outgoing branches.

Path: a path is a traversal of connected branches in the direction of the branch arrows.

The path should not cross a node more than once.

Open path: a open path starts at a node and ends at another node.

Closed path: closed path starts and ends at same node.

Forward path: it is a path from one input node to an output node that does not crass any node more than once.

Forward path gain: it is the product of the branch transmittance of forward path.

Individual loop: it is a closed path starting from a node and after passing through a certain part of a graph arrives at the same node without crossing any node more than once.

Loop gain: it is the product of the branch transmittances of a loop.

Non touching loops: if the loops does not have a common node, then they are said to be non touching loops.

Any 10 parameter (10)

V 2. Systems that can be represented by the same differential model but that are different physically are called analogous systems.

Force -- Voltage analogy

Mechanical system	Electrical system
Force F	Voltage e
Velocity V	Current I
Displacement X	Charge q
Frictional coefficient B	Resistance R
Mass M	Inductance L
Stiffness of spring K	Inverse of capacitance 1/C

VI 1. Over all gain $T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$ (5)

where T - Transfer function

P_K - Forward path gain of k^{th} forward path

Δ = 1 - (Sum of individual loop gains)
+ {Sum of gain product of all possible combination of two non touching loops}
- {Sum of gain product of all possible combinations of three non touching loops}
+ - ..

Δ_K - Δ for that part of the graph which is not touching k^{th} F.P

(2)

There are 3 forward paths $\therefore K = 3$

$P_1 = G_1 G_2 G_3 G_6$

$P_2 = G_1 G_2 G_4 G_6$

$P_3 = G_1 G_2 G_5 G_6$ (2)

Individual loops

$L_1 = -G_1 H_1$

$L_2 = -G_1 G_2 G_3 H_2$

$L_3 = -G_1 G_2 G_4 H_2$

$L_4 = -G_1 G_2 G_5 H_2$ (2)

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4)$ (10)

$\Delta_1 = 1 \quad \Delta_2 = 1 \quad \Delta_3 = 1$

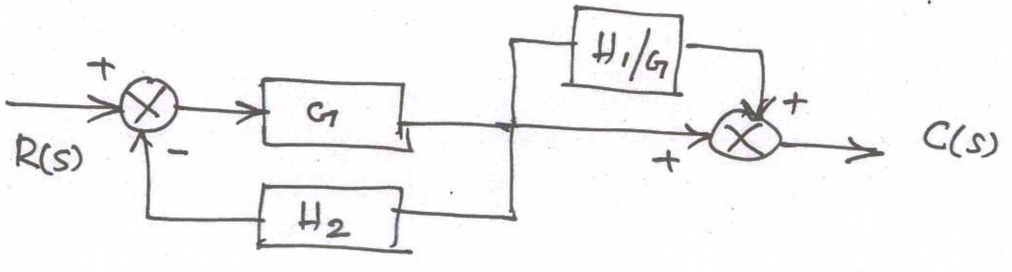
$\frac{C}{R} = \frac{1}{\Delta} \sum_K P_K \Delta_K$ here $K = 3$

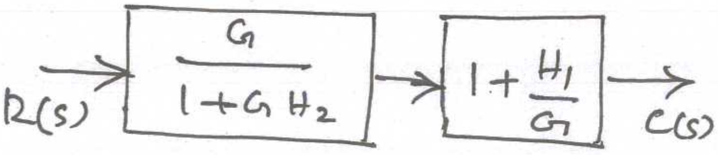
$= \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$

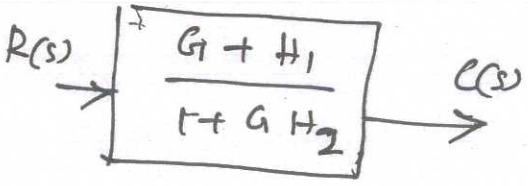
$= \frac{G_1 G_2 G_3 G_6 + G_1 G_2 G_4 G_6 + G_1 G_2 G_5 G_6}{1 + G_1 H_1 + G_1 G_2 G_3 H_2 + G_1 G_2 G_4 H_2 + G_1 G_2 G_5 H_2}.$

(4)

VI 2. Shift the take off point beyond block G.







(5)

VII 1. $s^5+6s^4+3s^3+2s^2+1=0$

$$\begin{array}{llll} s^5 & 1 & 3 & 1 \\ s^4 & 6 & 2 & 1 \\ s^3 & 2.67 & 0.83 & \\ s^2 & 0.135 & 1 & \\ s^1 & -18.95 & & \\ s^0 & 1 & & \end{array}$$

No. of sign changes in first column = 2

No. of poles on RHS = 2

Hence, system is unstable.

(8)

VII 2. The Loop transfer function of first order system is given by

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

For a unit impulse input $R(s) = 1$

$$C(s) = 1 \cdot \frac{1}{Ts+1} \qquad c(t) = L^{-1}\frac{1}{Ts+1} = L^{-1}\frac{1}{T(s+\frac{1}{T})}$$

$$= \frac{1}{T}e^{-t/T}$$

T is the system time constant. $\frac{1}{T}e^{-t/T}$ is the transient term $c_t(t)$ while the steady state term $c_{ss}(t) = 0$.

(7)

VIII 1. steady state error

$$e_{ss} = \underset{s\to 0}{Lt}\ \frac{s\,R(s)}{1+G(s)H(s)}$$

when the input is unit step $R(s)=\frac{1}{s}$

$$\therefore e_{ss} = \underset{s\to 0}{Lt}\ \frac{s\cdot\frac{1}{s}}{1+G(s)H(s)} = \frac{1}{1+K_p}$$

where $K_p = \lim_{s \to 0} G(s)H(s)$

The constant K_p is called positional error constant

Type 0 system

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$= \lim_{s \to 0} \frac{K(s+Z_1)(s+Z_2)\ldots}{(s+P_1)(s+P_2)\ldots}$$

$$= \frac{K \cdot Z_1 \cdot Z_2 \cdot Z_3 \ldots}{P_1 \cdot P_2 \cdot P_3 \ldots} = \text{constant.}$$

$$\therefore e_{ss} = \frac{1}{1+K_p} = \text{constant}$$

For type 1 and higher systems

$$K_p = \lim_{s \to 0} \frac{K(s+Z_1)(s+Z_2)\ldots}{s^N(s+P_1)(s+P_2)\ldots} = \infty$$

$$\therefore e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

Hence in type 0 systems when the input is unit step, there will be a constant steady state error.

In systems with type number 1 and above, for unit step input the value of K_p is ∞. and so the steady state error is zero.

VIII. 2.

The Routh stability criteria are an algebraic method that provides information on the absolute stability of a linear time invariant system with constant coefficients.

For stability of a closed loop system no roots of characteristic equation should be on the RHS of s-plane.

The procedure in Routh's stability criterion is as follows.

1. Write the characteristic equation in the following form,

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots a_{n-1} s + a_n = 0$$

The necessary but not sufficient condition to be satisfied for all the roots in the RHS of s plane is

1. All the coefficients should be of the same sign.

2. None of the coefficients should vanish.

If any of the coefficients are zero or negative, there is a root or roots which are imaginary or which have positive real parts. For such a case the system is not stable and for absolute stability, there is no need to follow the procedure.further. (7)

3. If all coefficients are positive, arrange the coefficients of the polynomial in rows and columns according to the following pattern.

$$\begin{array}{llllll}
s^n & : & a_0 & a_2 & a_4 & a_6 & \cdots \\
s^{n-1} & : & a_1 & a_3 & a_5 & a_7 & \cdots \\
s^{n-2} & : & b_0 & b_1 & b_2 & b_3 & \\
s^{n-3} & : & c_0 & c_1 & c_2 & & \\
\vdots & & & & & & \\
s^1 & : & g_0 & & & & \\
s^0 & : & h_0 & & & &
\end{array}$$

$$b_0 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_1 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$c_0 = \frac{b_0 a_3 - a_1 b_1}{b_0} \cdots$$

This row is continued until the nth row has been completed.

The Routh criterion states that the system represented by the array is stable if there is no sign change in the first column of the array. The number of sign changes equals the number of roots with positive real parts.

IX 1. Gain cross over frequency: it is the frequency at which the magnitude of the open loop transfer function is unity.

Phase cross over frequency: it is the frequency at which the phase of the open loop transfer function is 180^0.

Gain margin: the gain margin is defined as the reciprocal of the magnitude of open loop transfer function at phase cross over frequency.

Phase margin: the phase margin is that amount of additional phase lag at the gain cross over frequency, required to bring the system to the verge of instability.

$2 \times 4 = 8$

IX 2.

1. Locate the poles and zeros of G(s)H(s) on the s-plane. The root locus branch start from open loop ploes and terminate at zeros.

2. Determine the root locus on real axis. For a test point, if the total number of poles and zeros on the real axis to the right of the test point is odd number the test point lies on the root locus. If it is even then the test point does not lie on the root locus.

3. Determine the asymptotes of root locus branches and meeting point of asymptotes with real axis.

If n=no. of poles and m= no. of zeros.

Then (n-m) root locus branches will terminate at zeros at infinity. These root locus branches will go along an asymptotic path and meets the asymptotes at infinity. Hence number of asymptotes is equal to number of root locus branches going to infinity. The angles of asymptotes and the centroid are given by

Angles of asymptotes $=\frac{\pm 180(2q+1)}{n-m}$

Where q=0,1,2,3….(n-m)

Centroid (meeting point of asymptotes with real axis) $=\frac{sum\ of\ poles - sum\ of\ zeros}{n-m}$

4. breakaway and breakin points of the root locus are determined from the roots of the equation dK/ds =0. The roots of dK/ds =0 are actual breakaway or break in point provided for this value of root the gain K should be positive and real.

If there is a root locus on real axis between 2 poles then there exist a breakaway point. If there is a root locus on real axis between 2zeros then there exist a breakin point. If there is a root locus on real axis between pole and zero then there may be or may not be breakaway or breakin point.

5. if there is a complex pole then determine the angle of departure from the complex pole. If there is a complex zero then determine the angle of arrival at the complex zero.

Angle of departure (from complex pole A) = 180^0 – (sum of angles of vector to the complex pole A from other poles)+ (sum of angles of vector to the complex pole A from other zeros)

Angle of arrival at a complex zero A = 180^0 – (sum of angles of vectors to the complex zero A from other zeros)+ (sum of angles of vector to the complex zeroA from poles)

6. point of intersection of root locus with imaginary axis can be determined by use of the Routh criterion or by letting s=jω in the characteristic equation and equating the real part and imaginary part to zero, to solve for ω and K. The value of ω is the intersection point on imaginary axis and K is the value of gain at the intersection point.

7. Take a series of test point in the broad neighbourhood of the origin of the s plane and adjust the test point to satisfy angle criterion. Sketch the root locus by joining the test point by smooth curve. In practice the approximate root locus can be sketched from the informations obtained in steps 1 through 6 and from the knowledge of typical sketches of root locus. (7)

X 1. $G(s) = \frac{1}{1+sT}$

$G(j\omega) = \frac{1}{1+j\omega T} = \frac{1}{\sqrt{1+\omega^2T^2}} \angle -\tan^{-1}\omega T$

$|G(j\omega)|$ in db $= 20\log\frac{1}{\sqrt{1+\omega^2T^2}} = -20\log\sqrt{1+\omega^2T^2}$

At very low frequencies $\omega T << 1$

$|G(j\omega)| = -20\log\sqrt{1+\omega^2T^2} = -20\log 1 = 0$db

At very high frequencies $\omega T >> 1$

$|G(j\omega)| = -20\log\sqrt{1+\omega^2T^2} = -20\log\sqrt{\omega^2T^2}$

$= -20\log\omega T$ db

at $\omega = \frac{1}{T}$, $|G(j\omega)| = -20\log 1 = 0$db

$\omega = \frac{10}{T}$, $|G(j\omega)| = -20\log 10 = -20$db

using these approximations the magnitude plot of the factor $\frac{1}{1+j\omega T}$ can be approximated by two straight lines, one is a straight line at 0db for the frequency range $0 < \omega < \frac{1}{T}$

and the other is a straight line of slope -20db/decade for frequency range $\frac{1}{T} < \omega < \infty$. The two straight lines are asymptotes of the exact curve. The frequency at which the two asymptotes meet is called the corner frequency or break frequency.

The two lines intersect when

$$-20 \log 1 = -20 \log \omega_c T$$

$$\omega_c T = 1 \qquad \omega_c = \frac{1}{T}$$

ie for the factor $\frac{1}{1+j\omega T}$ the frequency $\omega_c = \frac{1}{T}$ is the corner frequency.

The actual magnitude at the corner frequency $\omega_c = \frac{1}{T}$ is

(4)

$$|G(j\omega)| \text{ in db} = -20 \log \sqrt{1+1} = -3\text{db}$$

Hence by this approximation the loss in db at the corner frequency is -3db

The phase angle due to $(1+j\omega T)^{-1}$ is

$$\phi = \angle(1+j\omega T)^{-1} = -\tan^{-1} \omega T$$

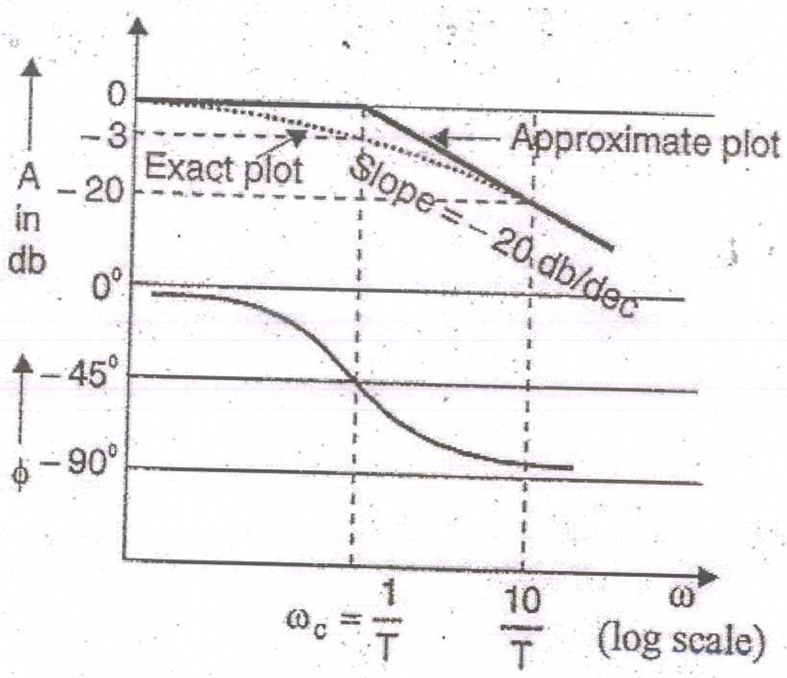
At corner frequency $\omega = \omega_c = \frac{1}{T}$

$$\phi = -\tan^{-1} 1 = -45^\circ$$

At $\omega \to 0$ $\phi = 0$ and at $\omega \to \infty$ $\phi = 90^\circ$

ie the phase angle of the factor $\frac{1}{1+j\omega T}$ varies from 0° to 90° as ω is varied from zero to infinity. The phase plot is a curve passing through -45° at ω_c.

(3)



(10)

(3)

Bode plot of the factor $\frac{1}{1+j\omega T}$.

X (2) 1. The magnitudes are expressed in db and so a simple procedure is available to add magnitude of each term one by one.

2. The approximate Bode Plot can be quickly sketched, and the corrections can be made at corner frequencies to get the exact plot.

3. The frequency domain specifications can be easily determined.

4. The Bode Plot can be used to analyse both open loop and closed loop system.

(5)