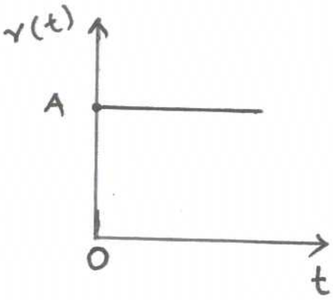
SCHEME OF VALUATION
(Scoring Indicators)

Revision : 2015 Course code: 5043

Course Title : CONTROL SYSTEMS

Qst No		Scoring Indicator	Split up Score	Sub Total	Total
		PART-A			
I	1	$\frac{1}{s^2}$	2	2	10
	2	$\frac{s+2}{(s+2)^2+9}$	2	2	
	3	Transfer function is the ratio of Laplace transform of output to laplace transform of input with zero initial condition	2	2	
	4	If the system is stable for all the range of system component values then the system is said to be absolutly stable system.	2	2	
	5	The path taken by a root of charecteristic equation when open loop gain K is varied from zero to infinity.	2	2	
		PART-B			
II	1	**Open loop system** — **Closed loop system** a) inaccurate and unrelable — Accurate and relable b) Feed back is absent — Feed back is present c) Stable — Unstable d) Simple and economical — Complex and costly e) Highly Sensitive to distorbancles — Less Sensitive f) Changes in o/p due to external distorbancles are not corrected automatically — Changes in output due to external distorbancles are corrected Automatically	6	6	
		a) Step Signal Signal whose values changes from zero to A at $t=0$ and remains constant at A for $t>0$			

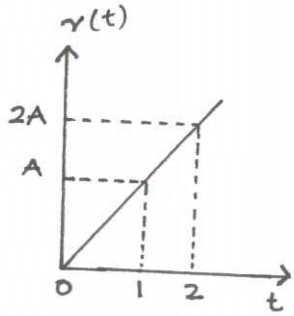
$$r(t) = A \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$



b) <u>Ramp Signal</u>

Signal whose values increases linearly with time from an initial value of zero at $t=0$

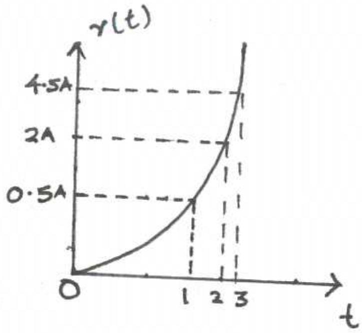
$$r(t) = At \; ; \; t \geq 0$$
2 $$= 0, \; ; \; t < 0$$ 3



c) <u>Parabolic Signal</u>

Signal in which the instantanious value varies as square of the time from an initial value of zero at $t=0$

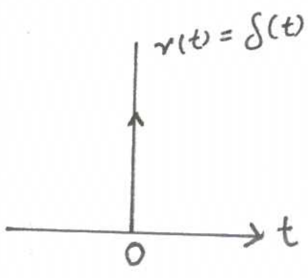
$$r(t) = \frac{At^2}{2} \; ; \; t \geq 0$$
$$0 \; ; \; t < 0$$



d) Impulse Signal

A signal of very large magnetude which is available for very short duration

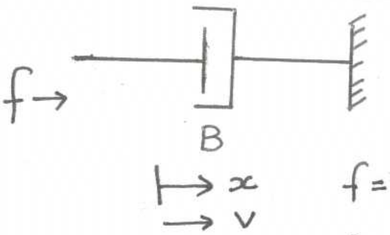
$$\delta(t) = \alpha \; ; \; t = 0$$
$$= 0 \; ; \; t \neq 0$$



3 6

Force - Voltage Analogy

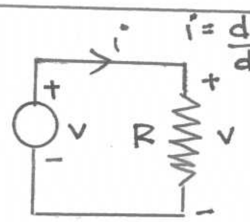
Mechanical System	Electrical system
input : Force	input : Voltage source
output : Velocity	output : current



$v = \frac{dx}{dt}$

$f = B\frac{dx}{dt}$

$f = Bv$

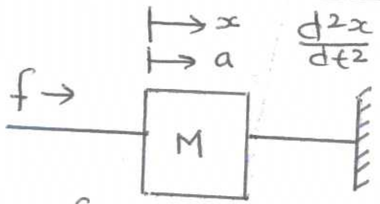


$i = \frac{dq}{dt}$

$e = v$

$v = Ri$

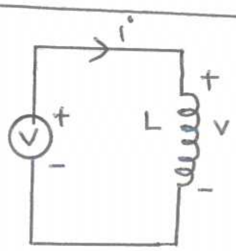
B analogous to R



$\frac{d^2x}{dt^2} = \frac{dv}{dt}$

$f = M\frac{d^2x}{dt^2}$

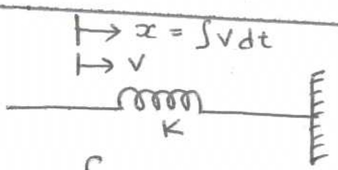
$f = M\frac{dv}{dt}$



$v = L\frac{di}{dt}$

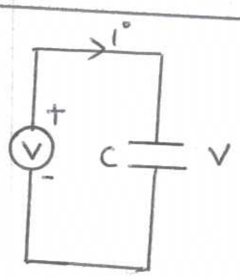
$v = L\frac{di}{dt}$ 3

3 'M' analogous to 'L'



$f = Kx$

$f = K\int v\,dt$

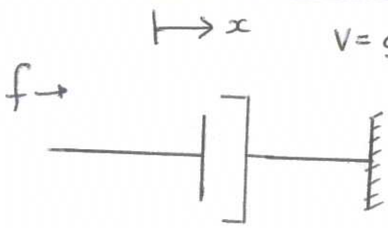


$v = \frac{1}{c}\int i\,dt$

'K' analogous to '$\frac{1}{c}$'

Force - Current Analogy

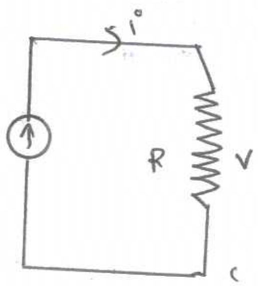
Mechanical system	Electrical system
input : Force	input : current
Output : Velocity	output : Voltage



$v = \frac{dx}{dt}$

$f = B\frac{dx}{dt}$

$f = Bv$



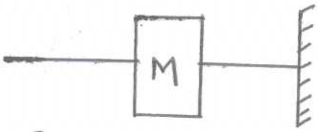
$i = \frac{v}{R}$

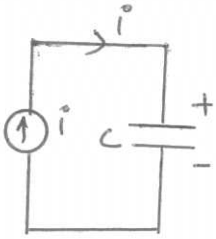
$i = \frac{1}{R}\cdot v$ 3

'B' analogous to '$\frac{1}{R}$'

$\mapsto x$

$\mapsto a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$





$i = C\frac{dv}{dt}$

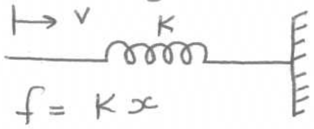
$f = M\frac{d^2x}{dt^2}$

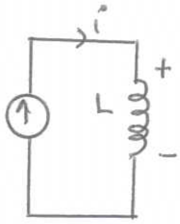
$f = M\frac{dv}{dt}$

'M' analogous to 'C'

$\mapsto x = \int v\,dt$

$\mapsto v$





$i = \frac{1}{L}\int v\,dt$ 6

$f = Kx$

$f = K\int v\,dt$

K analogous to $\frac{1}{L}$

4 (a) Transmittance :- The gain acquired by the signal when it travels from one node to another 2

(b) Individual loop :- Closed path starting from a node and ending at the same node and it does not cross any node more than once 2 6

(c) Forward path :- Path starts from input node and ends at output node which does not cross any node more than once 2

5 Charecteristic equation $s^4+3s^3+3s^2+2s+1=0$

s^4 :	1	3	1
s^3 :	3	2	0
s^2 :	7/3	1	0
s^1 :	5/7	0	0
s^0 :	1	0	0

4 6

- Location of roots -: All roots are laying on left half of 's' plane 1
- The system is stable 1

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$

6

No. of poles $P = 3$ $\quad P - Z = 3$

No. of Zeros $Z = NIL$

poles = $0, -2, -4$

Centroid $= \dfrac{\text{Sum of poles} - \text{Sum of Zeros}}{P-Z}$

$= \dfrac{0-2-4 - 0}{3} = \dfrac{-6}{3} = \underline{\underline{-2}}$ 3

Angle of asymptotes $= \dfrac{(2q+1)180^\circ}{P-Z}$ $\quad q = 0, 1, 2$

$= \dfrac{(2\times 0+1)180^\circ}{3} = 60^\circ$

$\dfrac{(2\times 1+1)180^\circ}{3} = 180^\circ$ 6

$\dfrac{(2\times 2+1)180^\circ}{3} = 300^\circ$ or -60° 3

Centroid $= -2$

Angle of asymptotes are $60^\circ, 180^\circ, 300^\circ$

7

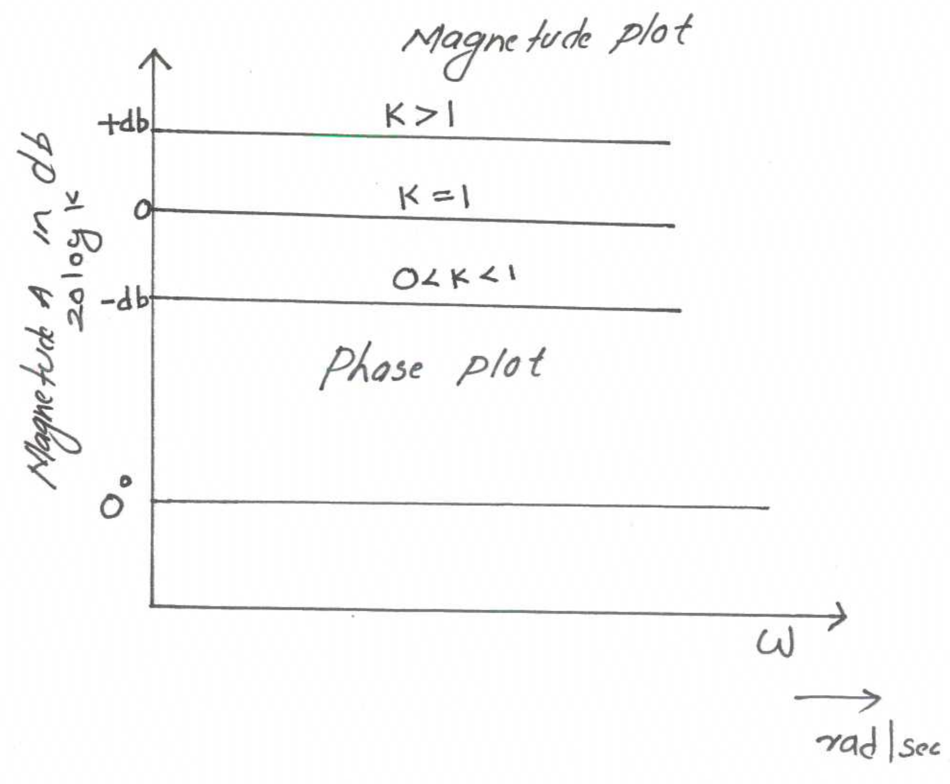
Let $G(s) = K$

$G(j\omega) = K = K\angle 0^\circ$

Magnetude $A = |G(j\omega)|$ in db $= 20\log K$

Phase angle $\phi = \angle G(j\omega) = 0^\circ$ 3

30



6

3

PART - C

III a

(i) Cos at

$$L\{f(t)\} = \int_0^{\alpha} e^{-st} f(t)\, dt$$

$$L\{\cos at\} = \int_0^{\alpha} e^{-st} \cos at\, dt$$

We have $\int e^{at} \cos bt = \frac{e^{at}}{a^2+b^2}\left[a\cos bt + b\sin bt\right]$ 2

By comparing $a = -s,\ b = a$

$$= \left[\frac{e^{-st}}{s^2+a^2}\left(-s\cos at + a\sin at\right)\right]_0^{\alpha}$$

$$= \frac{e^{-\alpha}}{s^2+a^2}\left[-s\cos\alpha + a\sin\alpha\right] - \frac{e^{-0}}{s^2+a^2}\left[-s\cos 0 + a\sin 0\right]$$

15

$$L\{\cos at\} = \frac{s}{s^2+a^2}$$

ii) e^{at}

6

$$L\{e^{at}\} = \int_0^{\alpha} e^{-st} \cdot e^{at}\, dt$$ 2

$$= \int_0^{\alpha} e^{-t(s-a)}\, dt$$

$$= \left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^{\alpha} = \frac{1}{s-a}$$

iii) 't'

$$L\{t\} = \int_0^{\alpha} e^{-st} \cdot t\, dt$$

$$= t \cdot \int e^{-st} - \int\left[\frac{d}{dt}(t) \cdot \int e^{-st}\right]$$ by product rule

$$= t \cdot \frac{e^{-st}}{-s} - \int 1 \cdot \frac{e^{-st}}{-s}$$ 2

$$= \left[\alpha \frac{e^{-\alpha}}{s} - \frac{1}{s^2} e^{-\alpha}\right] - \left[0 \cdot \frac{e^{0}}{s} - \frac{1}{s^2} \cdot e^{-0}\right]$$

$$= \frac{1}{s^2}$$

III b

(b)
$$\frac{2s^2-4}{(s+1)(s-2)(s-3)}$$

Let $\dfrac{2s^2-4}{(s+1)(s-2)(s-3)} = \dfrac{A}{s+1} + \dfrac{B}{s-2} + \dfrac{C}{s-3}$ 2

$$2s^2-4 = A(s-2)(s-3) + B(s+1)(s-3) + C(s+1)(s-2)$$

put $s=-1$ $\quad A = \dfrac{-1}{6}$

put $s = 2$ $\quad B = -4/3$ 3

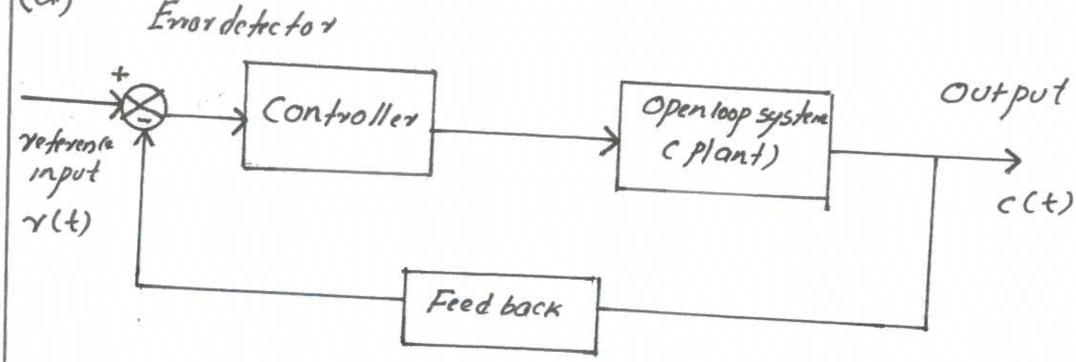
put $s=3$ $\quad C = 7/2$

$$\frac{2s^2-4}{(s+1)(s-2)(s-3)} = \frac{-1}{6(s+1)} - \frac{4}{3(s-2)} + \frac{7}{2(s-3)}$$ 9

$$= -\frac{1}{6}e^{-t} - \frac{4}{3}e^{2t} + \frac{7}{2}e^{3t}$$ 4

IV (a)

(a)



3 15

- Control system in which the output has an effect up on input quandity in order to Maintain the desired output valu are called closed loop system
- The feed back automatically corrects the change in output due to disturbances. Hence closed loop system also called Automatic control system 6
- closed loop system consist of an error detector, a controller, plant and feed back element 3
- The refference Signal (i/p signal) corresponds to desired output.
- The feed back path elements samples the output and convert it to a signal of same type as that of reference signal

The error Signal generated by error detector is the difference between reference Signal and feed back Signal.

- Controller modifies and amplifies the error Signal
- Modified error Signal is fed to the plant to correct its output.

IV b

(b)

$$L\{y\} = Y(s)$$

$$L\{y'\} = sY(s) - y(0)$$

$$L\{y''\} = s^2Y(s) - y(0) - y'(0)$$

$$L\{y'''\} = s^3Y(s) - y(0) - y'(0) - y''(0)$$

$$L\{e^{2t}\} = \frac{1}{s-a}$$ 2

Given $y'' - y = e^{2t}$

$$L\{y'' - y\} = L\{e^{2t}\}$$

$$[s^2y(s) - y(0) - y'(0)] - Y(s) = \frac{1}{s-2}$$

$$Y(s) = \frac{1}{(s-2)(s+1)}$$ 9

By partial fraction

$$\frac{1}{(s-2)(s+1)} = \frac{A}{(s-2)} + \frac{B}{(s+1)}$$ 3

$A = -1$

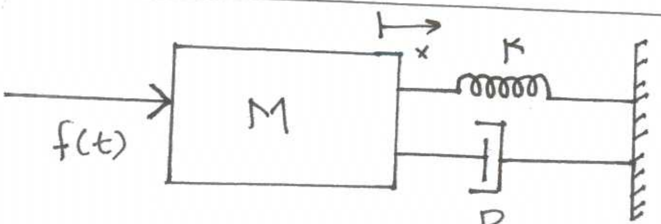
$B = -1/3$

$$\frac{1}{(s-2)(s+1)} = \frac{1/3}{(s-2)} + \frac{-1/3}{(s+1)}$$

$$= \frac{1}{3}e^{2t} - \frac{1}{3}e^{-t}$$ 4

V a

(a)



2 15

Applied force $= f(t)$

Total opposing force $= f_m + f_b + f_K$ 1

$$f_m = M\frac{d^2x}{dt^2}, \quad F_b = B\frac{dx}{dt}, \quad f_k = Kx$$

New ton's 2nd law $f(t) = f_m + f_b + f_k$

$$= M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$ 1

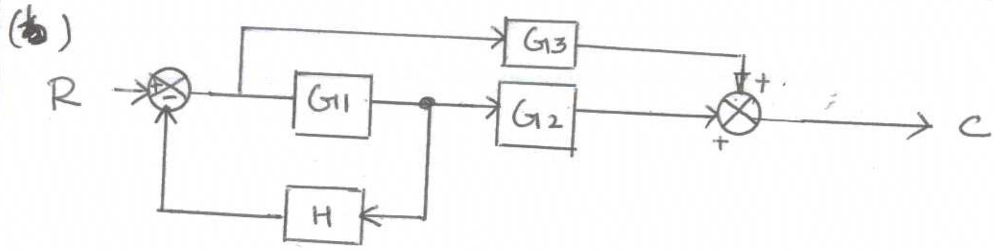
Taking L.T both Side

$$F(s) = X(s)\left[Ms^2 + Bs + K\right]$$ 6

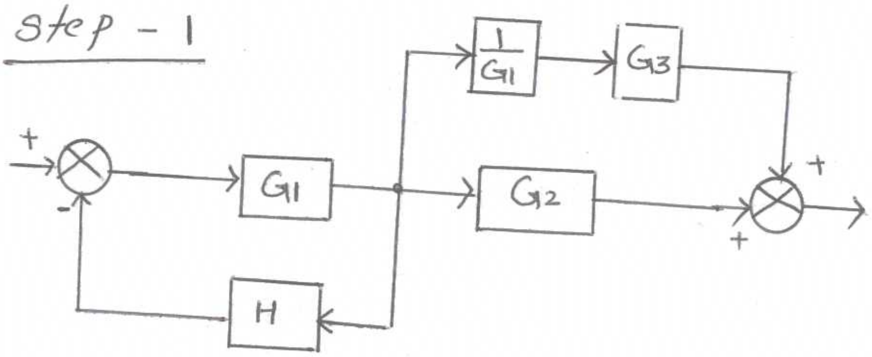
$$T.F = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$ 2

V b

(b)

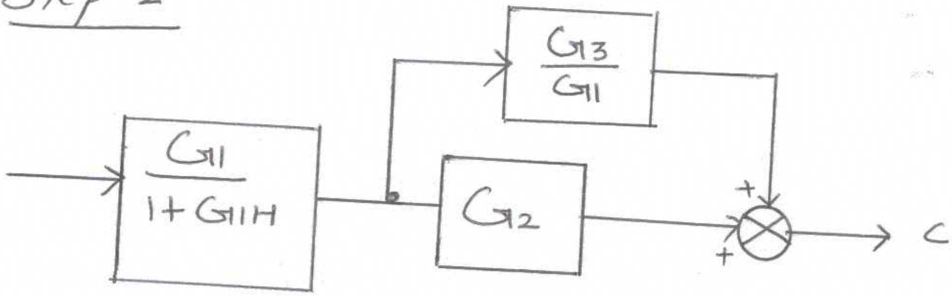


step - 1



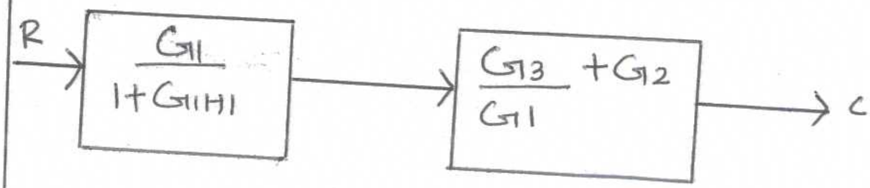
2

Step 2



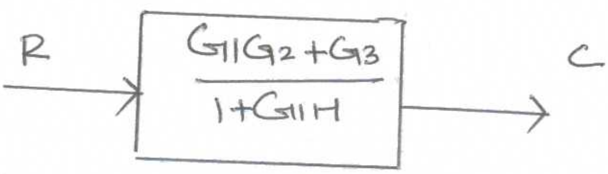
2 9

step - 3



2

Step - 4

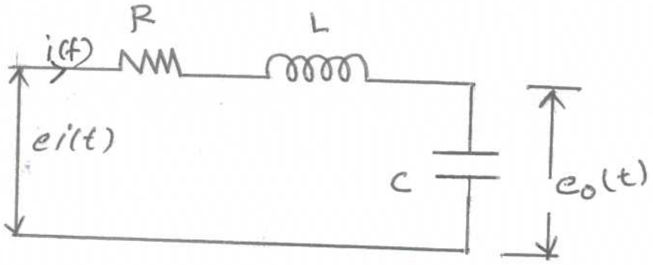


3

$$\frac{C}{R} = \frac{G_1G_2 + G_3}{1 + G_1H}$$

VI (a)

(a)



1

here $V_2 = e_o(t)$

Applay KVL $\quad e_i(t) = i(t)R + L\dfrac{di(t)}{dt} + \dfrac{1}{C}\int i(t)\,dt$

$$E_i(s) = I(s)R + Ls\,I(s) + \frac{1}{cs} I(s)$$

$$E_i(s) = I(s)\left[R + Ls + \frac{1}{cs}\right] - (1)$$

$$e_o(t) = \frac{1}{c}\int i(t)\,dt$$

6

$$E_o(s) = \frac{1}{cs} I(s)$$

4

$$I(s) = \frac{E_i(s)}{R + Ls + \frac{1}{cs}}$$

$$E_o(s) = \frac{E_i(s)}{Rcs + Lcs^2 + 1}$$

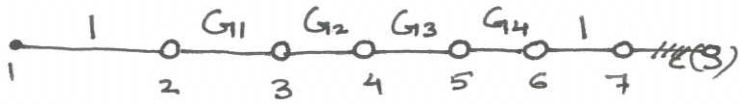
$$TF = \frac{1}{Rcs + Lcs^2 + 1}$$

1

VI (b)

(b) Forward path gain

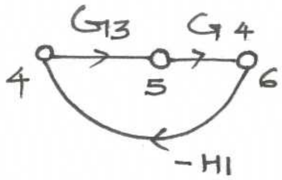
Only one forward path $K = 1$



1

Gain $= G_1 G_2 G_3 G_4$

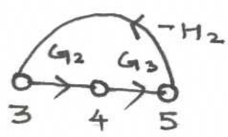
individual loop Gain



$P_{11} = -G_3 G_4 H_1$

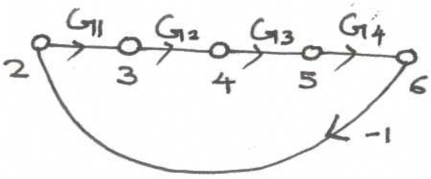
1

15



$P_{21} = -G_2 G_3 H_2$

1



$P_{31} = -G_1 G_2 G_3 G_4$

1

- No possible combination of two non touching loops three non touching loops etc

$\Delta = 1 - (P_{11} + P_{21} + P_{31})$

$= 1 + G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4$

1

$\Delta_1 = 1$

Transfer function T

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_{K} P_K \Delta_K$$ 2

$$= \frac{1}{\Delta} P_1 \Delta_1 \qquad \therefore K = 1$$ 9

$$= \frac{G_1 G_2 G_3 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4}$$ 2

VII a

(a) $H(S) = 1$

$$K_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{10(s+2)}{s^2(s+1)} \times 1 \left(\frac{1}{0}\right) = \alpha$$ 2

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \lim_{s \to 0} s\,\frac{10(s+2)}{s^2(s+1)} = \alpha$$ 2 6

$$K_a = \lim_{s \to 0} s^2\,G(s)H(s) = \lim_{s \to 0} \cancel{s^2}\,\frac{10(s+2)}{\cancel{s^2}(s+1)}$$

$$K_a = \frac{10 \times 2}{1} = \underline{\underline{20}}$$ 2

VII b

(b)

s^6	1	8	20	16
s^5	1	6	8	0
s^4	1	6	8	
s^3	0	0		
s^3	1	3		
s^2	3	8		
s^1	0.33	0		
s^0	8			

5 9 15

Auxiliary equation

$$A = s^4 + 6s^2 + 8$$ 2

$$\frac{dA}{ds} = 4s^3 + 12s$$

- No sign change 1
- System is marginally stable 1

VIII a (6 marks)

$r(t) = t \quad , \quad R(s) = \frac{1}{s^2}$

$C(s) = \frac{1}{1+sT} R(s)$

put $R(s) = \frac{1}{s^2}$ — 2

$C(s) = \frac{1}{s^2(1+sT)}$

By partial fraction

$C(s) = \frac{1}{s^2(1+sT)} = \frac{As+B}{s^2} + \frac{C}{sT+1}$ — 2

$A = -T$
$B = 1$
$C = T^2$

$\therefore \frac{-Ts+1}{s^2} + \frac{T}{sT+1}$

$L^{-1}\left\{\frac{-Ts+1}{s^2} + \frac{T}{sT+1}\right\}$ — 2

$= -T + t + Te^{-t/T}$

VIII b

Steady State error :- It is the value of e(t) when t tends to infinity — 2

$\therefore e_{ss} = \underset{t \to \alpha}{Lt}\ e(t)$

$e_{ss} = \lim_{s \to 0} \frac{s\,R(s)}{1+G(s)H(s)}$

For unit step input $R(s) = \frac{1}{s}$

$e_{ss} = \lim_{s \to 0} \frac{s \times 1/s}{1+G(s)H(s)} = \frac{1}{1+K_p}$ — 1

$K_p \rightarrow$ Positional error Constant

Type-0 system

$K_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} K \frac{(s+z_1)(s+z_2)(s+z_3)}{(s+p_1)(s+p_2)(s+p_3)}$ — 2

$= K \frac{z_1 z_2 z_3}{p_1 p_2 p_3} = \text{Constant}$

$e_{ss} = \frac{1}{1+K_p} = \text{Constant}$

Type - 1 system

$$K_P = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{K(s+z_1)(s+z_2)(s+z_3)}{s(s+P_1)(s+P_2)(s+P_3)}$$ 2

$$e_{ss} = \frac{1}{1+K_P} = \frac{1}{1+\alpha} = \underline{\underline{0}}$$

9 | 15

Type 2 system

$$K_P = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{K \cdot (s+z_1)(s+z_2)(s+z_3)}{s^2(s+P_1)(s+P_2)(s+P_3)}$$ 2

$$e_{ss} = \frac{1}{1+K_P} = \frac{1}{1+\alpha} = 0$$

IX a

$$\text{Magnetude } A = |G(j\omega)| = 20\log\left(\frac{K}{\omega}\right)$$

$$\phi = \angle G(j\omega) = -90^\circ$$

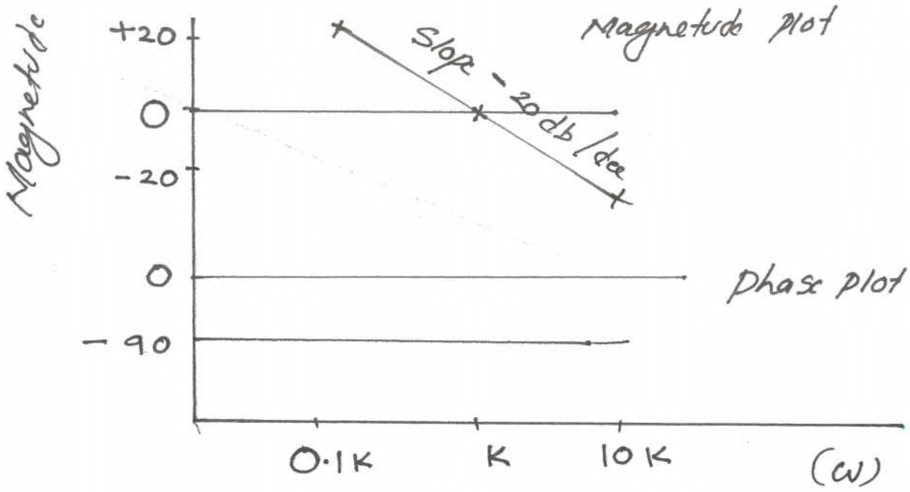
when $\omega = 0.1K$, $A = 20\log\left(\frac{1}{0.1}\right)$

$= 20\ db$

$\omega = K \qquad A = 20\log\frac{K}{K} = 0\ db$ 3

$\omega = 10K \qquad A = 20\log\frac{K}{10K} = -20\ db$

6 | 15



3

IX b

Step 1 -: Locate the Poles and zeros of $G(s)H(s)$ on the s plane

Step 2 -: Determine root locus on real axi's

Step 3 -: Determine Asymptotes of root locus and meeting point with real axi's

Step 4 -: Find break away and break in points

Step 5 -: Find the angle of departure and angle of arrival

Step 6 -: Find the crossing point of root locus crosses the imaginary axi's

Step 7 -: Sketch the root locus by joining the test point by smooth curve 9

X a

$\frac{K}{s(s+4)}$, $H(s) = 1$

poles $= 0, -4$, Zeros $=$ NIL

- Centroid $= \frac{\Sigma p - \Sigma z}{p - z} = \frac{0 - 4 - 0}{2} = -2$ 1

- Angle of Asymptotes $= \frac{(2q+1)180^\circ}{p-z}$ $q = 0, 1$ 1

$q = 0$, 90°

$q = 1$, 270° or -90° 15

- Break away point $= 1 + \frac{K}{s(s+4)} \times 1$ 1

$= \frac{s(s+4) + K}{s(s+4)} = 0$

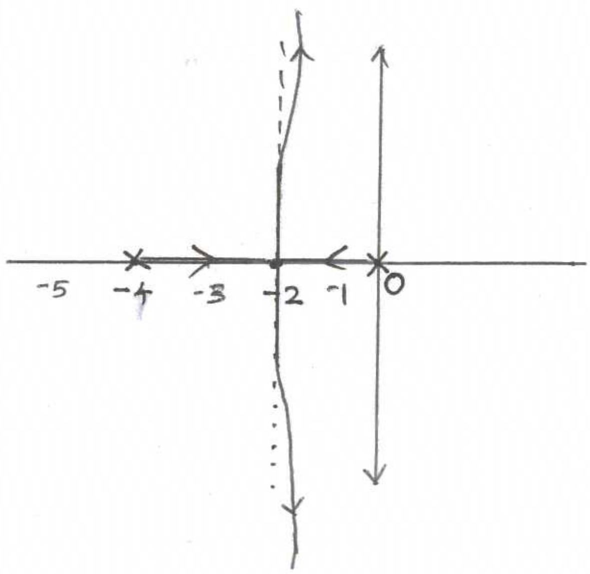
$s^2 + 4s + K = 0$

$K = -(s^2 + 4s)$

$\frac{dK}{ds} = -(2s+4) = 0$ 6

$\therefore$ $s = \underline{\underline{-2}}$ (break away point)

3



X	b	(i) Gain Margine —: The value by which the gain of the system has to be increased to drive system to be Verge of instability. - It is given by the reciprocal of the magnetude of open loop transfer function at phase cross over frequency $Kg = \frac{1}{\|G(j\omega)\|_{\omega=\omega_{pc}}}$ and $Kg \text{ in db} = 20\log \frac{1}{\|G(j\omega)\|_{\omega=\omega_{pc}}}$ $= -20\log \|G(j\omega)\|_{\omega=\omega_{pc}}$ (ii) Phase Margine —: Amount of additional phase lag at ~~given~~ gain cross over frequency ω_{gc} required to bring the system to the verge of instability. It is $180 + \phi_{gc}$	5	
		(iii) Phase Cross over frequency —: Frequency at which the phase of the open loop transfer function is $180°$	2	9
		(iv) Gain Cross Over frequency —: Frequency at which Magnetude of the open loop transfer function is unity	2	